Unit vs. Ad Valorem Taxes under Revenue Maximization*

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PRELIMINARY DRAFT

Abstract

We compare unit and ad valorem commodity tax regimes under the "Leviathan hypothesis" that the government seeks to maximize tax revenue. We show that the ad valorem tax regime welfare-dominates the unit tax regime *if and only if* the economy exhibits "ad valorem under-shifting" in response to a change in the tax level. Under Cournot competition, the level of shifting depends entirely on whether demand is not too convex so that elasticity of demand is increasing in price. In a more general framework, with differentiated goods, the threshold level of convexity such that unit taxes welfare dominate ad valorem ones can be lower.

Keywords: Commodity Taxation, Unit Tax, Ad Valorem Tax, Tax Shifting, Pass-Through, Revenue-Maximizing Government, Imperfect Competition.

JEL Codes: D40; H20; H21; H71.

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1 Introduction

Indirect taxation and revenue extraction from benevolent governments in either unit (or specific) or ad valorem tax regimes is a venerable issue in public finance, dating back to Cournot (1838) and Wicksell (1896). Ever since the work of Suits and Musgrave (1953), we have known that an ad valorem regime welfare dominates a unit regime in a monopoly setting, as, for any unit tax, there is an ad valorem tax which leaves tax revenue unchanged and raises output. Extensions of this result to oligopoly settings were initiated by Delipalla and Keen (1992) in a conjectural variations framework with homogeneous products and subsequently generalized by Anderson, de Palma, and Kreider (2001b) to imperfectly competitive markets. They show that welfare-dominance of ad valorem with a benevolent government extracting a given revenue holds in both Cournot and differentiated-Bertand frameworks when firms have symmetric costs, but not always in differentiated ones if firms have different costs.[1]

However, the assumption that the government is a benevolent social planner only presents one of the two mains views of policy-making; and a natural extension of the above-mentioned analyses is to compare both tax regimes under the opposite view, when the government maximizes its revenue from taxation. Indeed, since the work of Brennan and Buchanan (1977, 1978) on public choice, governments can also be represented by Leviathans which aim at maximizing their revenues, if public actors are believed to be rational and driven by self-interest. This view has been widely embraced, for instance, in the context of tax competition between countries or local governments; see, e.g., Kanbur and Keen (1993), Wang (1999), Janeba (2000), Edwards and Keen (1996), Lockwood (2004), Akai, Ogawa, and Ogawa (2011), and Aiura and Ogawa (2013).[2] In addition, rent extraction by local

[1] A stronger result than welfare-dominance was proven by Skeath and Trandel (1994), who show that there always exists an ad valorem tax which is Pareto-dominant to any unit tax in a monopoly setting, and that this result holds for Cournot oligopolies under linear demand for some parameters. See also the paper by Peitz and Reisinger (2014) for comparison of ad valorem vs. unit tax regimes in vertically-related markets.

[2] Kanbur and Keen (1993) show that competition between tax-revenue-maximizing countries that set unit taxes lead to an inefficient outcome. This outcome is even worse when countries differ in size, with the smaller country charging a lower unit tax. Wang (1999) extends their analysis by allowing for the large country to set its unit tax level first, in a Stackelberg manner. Janeba (2000) studies the commitment problem of competing

governments, which is closely related to Leviathan-like behavior, has been documented by Brueckner and Neumark (2014) and Diamond (2014).

Some results from the above-mentioned literature on ad valorem vs. unit taxation can also be used under the Leviathan hypothesis. First, when the market is perfectly competitive, the equilibrium output would be equal to the monopoly one under both tax regimes, as firms charge no mark-up and the government acts as a monopolist which faces the whole demand curve. Second, the analysis of Suits and Musgrave (1953) implies that a revenue-maximizing government with control over the tax regime would always select the ad valorem one; and Delipalla and Keen (1992) and Anderson, de Palma, and Kreider (2001b) have demonstrated that this would also hold in oligopoly settings when firms have similar technologies.[3] These analyses give no insight, however, on welfare comparison between the two tax regimes with a Leviathan government.

In this paper, we analyze the welfare effects of a government's choice of tax level or rate when it maximizes its revenue. We show that total surplus is larger under the revenue-maximizing ad valorem tax rate than under the revenue-maximizing unit tax level if and only if the elasticity of (total) demand increases in price.[4] By contrast, when this condition

tax-revenue-maximizing governments which can change the level of their unit taxes after a firm has invested on their territories. Edwards and Keen (1996) compare both the social planner and Leviathan outcomes in the context of tax competition. Studying capital tax competition, Lockwood (2004) compares outcomes under both unit and ad valorem tax regimes when governments maximize tax revenues by competing in tax levels or rates, and Akai, Ogawa, and Ogawa (2011) extend his results by allowing governments to choose the tax regime in the first stage. Aiura and Ogawa (2013) provide a similar analysis in the spatial taxation context with cross-border shoppers, where revenue-maximizing governments compete at the tax levels or rates with endogenous choice of the tax regime.

[3]Some insights can also be taken from the industrial organization literature, where some authors have analyzed the outcome of different vertical contracts between a monopolist manufacturer and a monopolist retailer, when the contract is either a linear pricing or a revenue-sharing agreement. For instance, Gaudin and White (2014) compare so-called "wholesale" and "agency" vertical agreements between a publisher and a retailer in the electronic books market, and Llobet and Padilla (2014) compare per-unit and revenue-based royalties between a licensor and a licensee in the context of intellectual property rights. Both papers show that revenue-sharing contracts lead to a larger output if and only if the elasticity of demand increases in prices. Our results generalize this literature, by introducing imperfect competition at the retail level. Also see Shy and Wang (2011) and Wang and Wright (2014) for related work on payment systems.

[4]In the case of monopoly, the relationship between the derivative of the elasticity of demand and the relative output under unit and ad valorem regimes was first derived by Bishop (1968). This simple condition can be rewritten as a particular relationship between demand elasticity and curvature, which we detail in Section 3. Mràzovà and Neary (2013), who study how the interplay between these two parameters impacts some classical comparative statics results, call this condition on the demand form "subconvexity," and provide several examples of parametric families of demand functions satisfying it.

does not hold, total surplus is higher in a unit tax regime, whereas tax revenue is larger in an ad valorem tax regime. This result is robust to a wide set of demand forms and to various frameworks of imperfect competition with a constant conduct parameter, including monopoly, Cournot, and several models of price competition with differentiated products. We also extend the analysis to the more general case of non-constant conduct parameter, thereby including any setting of price competition with differentiated products.

This simple, clear-cut, result has important policy implications, in addition to understanding the relative distortionary effects of the two taxation regimes. It shows how, in the spirit of Brennan and Buchanan (1977, 1978) and Wilson (1989), the choice between ad valorem and unit taxation regimes could serve as a constitutional constraint, thereby *taming* a Leviathan government, when set by consumers or by a welfare-maximizing, higher-level (e.g., federal), government.

The remainder of the paper is as follows. Section 2 describes the basic assumptions of the Cournot mode. Equilibria in both unit and ad valorem regimes are derived and compared in Section 3. Section 4 extends the analysis to price competition and differentiated markets. Finally, Section 5 concludes. Proofs are relegated to the Appendix.

2 The model

This section describes the model, focusing on the symmetric Cournot setup, which we then generalize in Section 4. A government levies an indirect tax on a set of quantity-competing firms. Our goal is to compare the equilibrium outcomes arising under two different forms of taxation: unit and ad valorem. Under the former, the government sets a unit tax, τ, that each firm must pay on every unit sold. Under the latter regime, the government sets an ad valorem tax rate, t, corresponding to a percentage of each firm's retail price. Throughout this paper, we will refer to the ad valorem ratio $a \equiv t/(1+t)$, which is equivalent to the fraction of revenue captured by the government. We assume that the government does not incur any cost in collecting the tax.

Demand is given by the non-negative function $D(\cdot)$, defined on $(0, \infty)$ and assumed to be strictly decreasing and thrice differentiable wherever in it is strictly positive. The inverse demand function is denoted by $P(\cdot)$. There are n firms that compete in quantity, with $n \geq 1$. We denote by Q the total market quantity, and by q^i firm i's output, with $\Sigma_{i=1}^{n} q^i = Q$. All firms have the same constant marginal cost of production, $c \in (0, \overline{c})$, where $\overline{c}$ is sufficiently small to ensure positive output in equilibrium under both tax regimes.[5]

The timing is as follows. Under both the unit and ad valorem regimes, the government first chooses the tax rate. After observing this choice, the competing firms simultaneously set their quantities.

Throughout the paper we assume that the demand function satisfies the following condition globally (omitting arguments when no ambiguity arises).

Assumption 1 (Stability condition)**.** $2P' + QP'' < 0.$

Assumption 1 ensures the stability of the Cournot game for any number of firms, and it amounts to an assumption that the marketwide marginal revenue curve, $P + QP'$, is strictly decreasing.[6] This implies that each firm's marginal revenue decreases in its own output in a symmetric equilibrium, that is, $2P' + q^i P'' < 0$, which guarantees that each firm's profit function is quasi-concave in its own quantity.

3 Equilibria and comparison of both tax regimes

We now determine the equilibrium under each tax regime, restricting attention, throughout, to equilibria that are symmetric across firms. Let $MR(Q) \equiv P(Q) + QP'(Q)/n$ denote the marginal revenue per firm as a function of aggregate quantity, when firms behave symmetrically. Also, define $\overline{Q} > 0$ as the unique quantity level such that $\lim_{Q \to \overline{Q}} MR(Q) = 0$.

[5]When $c = 0$, the game is trivial in the ad valorem regime, because the revenue-percentage tax becomes a profit-sharing one and firms and government seek to maximize total revenue. In this case, in any subgame-perfect equilibrium, the government captures all revenue, and the firms' output level is not uniquely determined.

[6]For a fixed number of firms, n, the condition guaranteeing stability of the Cournot game is $(1 + 1/n) P' + qP'' < 0$ (see Seade (1980)).

(This encompasses the case where the marginal revenue crosses the horizontal axis, i.e., where $MR\left(\overline{Q}\right) = 0$, and the case where the curve asymptotes to zero.)

3.1 Equilibrium under unit taxation

In a unit taxation regime, the government first sets the unit tax level, τ, and then each firm sets its retail quantity. We solve the game backwards.

Quantity setting. In the last stage of the game, τ is fixed. Firm i's profit in the unit tax regime is denoted by $\pi_u^i\left(q^i\right) \equiv \left[P(Q) - \tau - c\right] q^i$. Optimal quantity setting requires that,

$$\begin{aligned} \forall i,\ \frac{\partial \pi_u^i(q_u^i)}{\partial q^i} = 0 \quad &\Leftrightarrow P(Q_u) - \tau - c + q_u^i P'(Q_u) = 0 \\ &\Leftrightarrow MR(Q_u) = \tau + c, \end{aligned} \tag{1}$$

where q_u^i is firm i's equilibrium quantity in the unit tax regime, and where $Q_u = nq_u^i$ by symmetry.

Tax setting. In the first stage of the game, the public authority maximizes its revenue from taxation, $T_u(\tau) \equiv \tau Q_u$. It sets $\tau = \tau^*$ in equilibrium, such that

$$\begin{aligned} \frac{\partial T_u(\tau^*)}{\partial \tau} = 0 \quad &\Leftrightarrow \tau^* \frac{\partial Q_u}{\partial \tau} + Q_u = 0 \\ &\Leftrightarrow \tau^* = -Q_u MR'(Q_u), \\ &\Leftrightarrow MR(Q_u) + Q_u MR'(Q_u) = c, \end{aligned} \tag{2}$$

where the last expression is derived by by plugging equation (1) into the government's maximization problem. Note that, by Assumption 1, $MR' = (1 + 1/n) P' + QP''/n < 0$ and $MR'' = (1 + 2/n) P'' + QP'''/n$. The left-hand side of the last expression, $MR + QMR'$, corresponds to the adjusted marginal revenue curve which intersects the marginal cost curve in equilibrium.

We define the elasticity of demand, $\varepsilon(Q) \equiv -P(Q) / [QP'(Q)]$, and the elasticity of the slope of inverse demand, $E(Q) \equiv -QP''(Q) / P'(Q)$. Noting that the derivative of the elasticity of demand with respect to the market quantity can be expressed as

$$\varepsilon'(Q) = \frac{-\varepsilon(Q)}{Q}\left(1 + \frac{1}{\varepsilon(Q)} - E(Q)\right), \tag{3}$$

the above-mentioned adjusted marginal revenue curve can be expressed as follows.

Definition 1. *Let*

$$U(Q) \equiv MR(Q)\left(1 - \frac{1}{\varepsilon(Q)} + \frac{Q}{\varepsilon(Q)}\frac{\varepsilon'(Q)}{(n\varepsilon(Q) - 1)}\right)$$

denote the "Unit tax-adjusted marginal revenue" curve.

To guarantee the existence of a unique solution to the government's problem, we impose the following second-order condition.

Assumption 2 (Government's SOC – Unit tax regime)**.** *$U'(Q) = 2MR'(Q) + QMR''(Q) < 0$, for all Q such that $U(Q) > 0$.*

We can reformulate the equilibrium given by equation (2) to state the following lemma.

Lemma 1. *The equilibrium in the unit tax regime is given by $U(Q) = c$. In addition, there exists a unique quantity $\hat{Q} \in \left(0, \overline{Q}\right]$ such that $\lim_{Q \to \hat{Q}} U(Q) = 0$, and $U(\cdot)$ is positive, continuously differentiable and strictly decreasing on $\left(0, \hat{Q}\right)$.*

3.2 Equilibrium under ad valorem taxation

In an ad valorem taxation regime, the government first sets the ad valorem tax ratio, a, and then each firm sets the retail quantity. As above, the game is solved backwards.

Quantity setting. In the last stage of the game, a is fixed. Firm i maximizes its profit in the ad valorem tax regime, $\pi_a^i\left(q^i\right) \equiv [(1-a)P(Q) - c]\,q^i$. Profit-maximization implies that,

$$\begin{aligned} \forall i,\ \tfrac{\partial \pi_a^i(q_a^i)}{\partial q^i} = 0 \quad &\Leftrightarrow (1-a)\left[P(Q_a) + q_a^i P'(Q_a)\right] - c = 0 \\ &\Leftrightarrow (1-a)\,MR(Q_a) = c. \end{aligned} \tag{4}$$

where q_a^i is firm i's equilibrium quantity in the ad valorem tax regime, and where $Q_a = nq_a^i$ by symmetry.

Tax setting. In the first stage, the government maximizes its tax revenue, $T_a(a) \equiv aP(Q_a)Q_a$. This gives first-order condition

$$\begin{aligned}
\frac{\partial T_a(a^*)}{\partial a} = 0 \quad & \Leftrightarrow a^* \frac{\partial Q_a}{\partial a}\left[P(Q_a) + Q_a P'(Q_a)\right] + Q_a P(Q_a) = 0 \\
& \Leftrightarrow a^* = \left(1 - \frac{MR(Q_a)[P(Q_a)+Q_aP'(Q_a)]}{P(Q_a)Q_aMR'(Q_a)}\right)^{-1} \\
& \Leftrightarrow \frac{MR^2(Q_a)[P(Q_a)+Q_aP'(Q_a)]}{MR(Q_a)[P(Q_a)+Q_aP'(Q_a)]-Q_aP(Q_a)MR'(Q_a)} = c,
\end{aligned} \tag{5}$$

where the last expression is derived by plugging equation (4) into the government's maximization problem, and where a^* denotes the equilibrium ad valorem tax ratio. As in the unit tax regime, we can reformulate the left-hand side of the last expression using the elasticity of demand.

Definition 2. *Let*

$$A(Q) \equiv \left(1 - \frac{1}{\varepsilon(Q)}\right) \frac{MR(Q)}{\left(1 - \frac{Q}{\varepsilon(Q)} \frac{\varepsilon'(Q)}{(n\varepsilon(Q)-1)}\right)}$$

denote the "Ad valorem tax-adjusted marginal revenue" curve.

We assume that the second-order condition of the government's problem is satisfied, which is as follows.

Assumption 3 (Government's SOC – Ad valorem tax regime). *$A'(Q) < 0$, for all Q such that $A(Q) > 0$.*

As above, we can reformulate the equilibrium given by equation (5), using the adjusted marginal revenue, to state the following lemma.

Lemma 2. *The equilibrium in the unit tax regime is given by $A(Q) = c$. In addition, there exists a unique quantity $\tilde{Q} \in \left[\hat{Q}, \overline{Q}\right]$ such that $\lim_{Q \to \tilde{Q}} A(Q) = 0$, and $A(\cdot)$ is positive, continuously differentiable and strictly decreasing on $\left(0, \tilde{Q}\right)$.*

3.3 Comparison of the two equilibria

We now compare the equilibrium outcomes obtained under both tax regimes. The subtlety of this exercise stems from the fact that, when the market is not perfectly competitive, equilibrium quantities will typically differ from one tax regime to the other. However, comparing both cases at their respective equilibria is equivalent to comparing both adjusted marginal revenue curves defined above, as they are continuous and monotonically decreasing over the relevant range due to the second order conditions.

To see this, note that the ranking of both adjusted marginal revenue curves remains stable over both equilibrium quantities, as stated in the following Lemma.

Lemma 3. *Let Q_u denote equilibrium output under the unit tax regime, and let Q_a denote equilibrium output under the ad valorem tax regime. It holds that* $sign\{A(Q_u) - U(Q_u)\} = sign\{A(Q_a) - U(Q_a)\}$.

Lemma 3 shows that it is irrelevant which of these quantities one uses when comparing the two adjusted marginal revenue curves compare. The intuition for this result is as follows. A given marginal cost curve intersects with both marginal revenue curves once, thus defining two equilibrium quantities. The adjusted marginal revenue curve that is found further right at this level of marginal cost leads to a larger equilibrium quantity. As this curve is monotonically decreasing, it stands above the curve that is further left at this marginal cost, when evaluated at the smallest of the two equilibrium quantities. By symmetry, the same ranking stands at the largest equilibrium quantity, as well as for any quantity in the interval bounded by both equilibrium quantities.[7] In the following, we simply refer to this interval, which may reduce to a single point, as *equilibrium quantities*.

We now need to compare both adjusted marginal revenue curves at equilibrium quantities. As explained in the following Lemma, this comparison depends on a simple relationship between the elasticity of the slope of inverse demand, E, and the elasticity of demand, ε.

[7]Note that this result also holds for non-constant marginal cost curves, as long as the equilibrium under each tax regime is unique.

Lemma 4. *For any quantity $Q \in \left(0, \overline{Q}\right)$, it holds that*

$$sign\left\{A\left(Q\right) - U\left(Q\right)\right\} = sign\left\{1 + 1/\varepsilon\left(Q\right) - E\left(Q\right)\right\}.$$

Now that we know how to compare both tax regime equilibrium quantities, for a given marginal cost, we can state our main result.

Proposition 1. *Under Cournot competition, when the government maximizes its tax revenue, equilibrium output under the ad valorem tax regime is greater than equilibrium output under the unit tax regime if and only if $E < 1 + 1/\varepsilon$ at equilibrium quantities.*

How outputs under both tax regimes compare thus depends on a simple relation involving demand elasticity and curvature, as measured by the elasticity of the slope of the inverse demand. Using the expression of the derivative of the elasticity of demand given by equation (3), we can restate this result as follows.

Corollary 1. *Under Cournot competition, when the government maximizes its tax revenue, equilibrium output under the ad valorem tax regime is greater than equilibrium output under the unit tax regime if and only if the elasticity of demand decreases in quantity (or increases in price) at equilibrium quantities.*

In our setting, the number of firms remains fixed, and, hence, the regime leading to the larger output also leads to greater total surplus. Therefore, the tax-revenue-maximizing ad valorem rate is included in the (non-empty) set of ad valorem rates that welfare-dominate the tax-revenue-maximizing unit tax level if and only if the equilibrium elasticity of demand increases in price (or decreases in quantity). Constant-elasticity demand forms therefore constitute a limit case for which the equilibrium quantity would always be the same under both taxation regimes, for any marginal cost, when the government maximizes tax revenues. By contrast, demand functions which do not display an increasing elasticity at all prices lead to a lower quantity in ad valorem than in unit tax regime at equilibrium, for marginal costs that induce the equilibrium price to fall into intervals where the elasticity decreases in price. Illustrating examples for these different cases are given, for the monopoly setting, in Appendix B.

The result that the unit tax regime can be welfare-enhancing as compared to the ad valorem tax regime under a Leviathan government may seem surprising at first. However, because there is always a welfare-dominant ad valorem tax rate to any unit tax level does not mean that this dominance holds when comparing revenue-maximizing equilibria. To understand this, consider first a switch from the revenue-maximizing unit tax to the ad valorem rate which keeps tax revenue constant after the firms have adjusted their output. This switch always increases output, because of the welfare-dominance of the ad valorem regime at a given tax revenue. Then, allow the government to increase its ad valorem tax rate to maximize its revenue, and the firms to adjust their output accordingly: this will decrease output. How severe is this output reduction depends on the curvature of demand, which can be evaluated through the slope of the elasticity.

Alfred Marshall referred to the elasticity of demand being increasing in price as the "law of the elasticity of demand" (Marshall, 1890, Book III, Chapter IV, §2, pp. 103-104). He stated that this "appears to hold with regard to nearly all commodities and with regard to the demand of every class." This "Marshall's Second Law of Demand" is equivalent to the pass-through rate an hypothetical monopolist with constant marginal cost would face, $dp/dc = 1/(2 + qP''/P')$, to be lower than the ratio $1/(1 - 1/\varepsilon)$, which corresponds to the pass-through rate from a constant-elasticity demand, and which lies above unity. Therefore, any demand form that is (locally) more convex than a constant-elasticity demand form – what Mràzovà and Neary (2013) refer to as "superconvexity" – will break Marshall's Second Law of Demand at equilibrium, for some given marginal costs.[8]

The condition on demand we highlight in Proposition 1 also relates to the incidence of an ad valorem tax. Indeed, in the Cournot framework, $E < 1 + 1/\varepsilon$ is equivalent to under-shifting of the ad valorem tax, which corresponds to the case where an increase in

[8] Note that commonly used demand forms satisfy Marshall's Second Law of Demand everywhere: Amir, Maret, and Troege (2004) demonstrate that log-concavity of demand is equivalent to a pass-through rate being smaller than unity, and Fabinger and Weyl (2014), building on early work by Bulow and Pfleiderer (1983), show that any constant pass-through rate lies (weakly) below $1/(1 - 1/\varepsilon)$.

the ad valorem tax leads to a decrease in producer prices, $(1-a)\,p$.[9] By contrast, when the elasticity of demand decreases in price, there is over-shifting of the ad valorem tax.

Under a Leviathan government, which of the two tax regimes leads to a larger total welfare is therefore an empirical question. Estimating tax incidence in (potentially imperfectly) competitive markets, Besley and Rosen (1999) find over-shifting of ad valorem taxes for more than half of the product samples they analyze. According to the above discussion, this could imply that the market demand form is convex enough to induce a unit tax regime to welfare-dominate an ad valorem one under the Leviathan hypothesis. In another study, Karp and Perloff (1989) estimate an over-shifting of an ad valorem tax in the oligopolistic Japanese television market. Others empirical analyses estimate pass-through rates that are large enough to conjecture that the elasticity of demand could increase with quantity. For instance, Besanko, Dubé, and Gupta (2005) focus on retail products sold in oligopoly markets and find that more than 30% of pass-through rates are larger than unity, with half of those even being larger than 1.5, thereby making these markets' demand forms potential candidates for being more convex that constant-elasticity ones.

Finally, note that our results are robust to non-constant, continuous and symmetric (across firms) marginal costs. Indeed, the above-mentioned equivalence between comparing equilibrium outputs under both regimes and comparing both adjusted marginal revenue curves also holds with non-constant marginal costs. This is true as long as a unique equilibrium exists under each tax regime, as the marginal cost curve intersects with the (locally) *lowest* adjusted marginal revenue curve first, thereby leading to a smaller output at equilibrium for the corresponding tax regime than for the other one.

[9] As for the comparison with the pass-through rate, this relies on the assumption of constant marginal costs. See Anderson, de Palma, and Kreider (2001a) for an analysis of tax incidence in oligopolies in both regimes, and Weyl and Fabinger (2013) for an extensive treatment in the unit regime. See also Fullerton and Metcalf (2002) for a review.

4 Imperfect competition with differentiated products

We now extend our analysis to the case of imperfect competition between firms selling differentiated products. In particular, we show that the condition on demand curvature for the unit tax regime to lead to a larger output than the ad valorem one is typically *weaker* in the case of price competition with differentiated products than for quantity competition with homogeneous products, as given in Proposition 1.

A symmetric equilibrium of the last-stage game in which n firms sell differentiated products, with the same constant marginal cost, c, in a market of size normalized to unity, can be written as:

$$P(q) + q\theta(q) P'(q) = \tilde{c}, \tag{6}$$

where q is the total number of consumers served by the market, $P(\cdot)$ is the inverse market demand, and where $\tilde{c} = c + \tau$ under a unit tax regime, and $\tilde{c} = c/(1 - a)$ under an ad valorem one.

The conduct parameter, $\theta \in [0, 1]$, can be interpreted as an index of market competitiveness, as explained by Bresnahan (1989) and Genesove and Mullin (1998). For instance, this parameter can be used to model perfect competition ($\theta = 0$), monopoly ($\theta = 1$), or homogeneous Cournot competition ($\theta = 1/n$). When it is constant and firms set quantities, this reduced-form model corresponds to a conjectural variation model, as studied by Delipalla and Keen (1992). However, allowing the conduct parameter and the quantity q to vary according to firms' strategic variables allows this model to represent frameworks of imperfect competition in which differentiated firms compete in prices, quantities, or supply functions *à la* Klemperer and Meyer (1989).[10] The equilibrium given by equation (6) thus nests any symmetric framework of imperfect competition, including, for instance, the logit model of price competition. Appendix C.1 provides a general definition of the conduct parameter, and specifications in the cases of price and quantity competition with differentiated products.

[10]Micro-foundations for this reduced-form model are given by Mahoney and Weyl (2014), with applications to the analysis of selection markets.

From equation (6), and following the same reasoning than in Section 3, we can derive the equilibrium under each taxation regime, when the government maximizes tax revenue. Details of the analysis are provided in Appendix C.2. Redefining a firm's marginal revenue, $\widehat{MR}(q)$, as the left-hand side of equation (6), we obtain the following adjusted marginal revenue curves, $\widehat{U}(\cdot)$ and $\widehat{A}(\cdot)$, under unit and ad valorem tax regimes, respectively, whose intersections with the marginal cost curve, c, determine the equilibria.

$$\begin{cases} \widehat{U}(q) \equiv & \widehat{MR}(q) + q\widehat{MR}'(q), \\ \widehat{A}(q) \equiv & \widehat{MR}^2(q)\left[P(q) + qP'(q)\right] / \left\{\widehat{MR}(q)\left[P(q) + qP'(q)\right] - qP(q)\widehat{MR}'(q)\right\}. \end{cases} \tag{7}$$

These adjusted marginal revenue curves are similar to that given by equations (2) and (5), with the only difference that the newly defined marginal revenue is given by equation (6), and that its derivative is $\widehat{MR}' = (1 + \theta + q\theta')P' + q\theta P''$, where θ' is the derivative of the conduct parameter with respect to quantity and is assumed to exist and to be differentiable everywhere. Assuming that the marketwide marginal revenue is strictly decreasing and that second-order conditions are satisfied (see Appendix C.2), we can compare these curves the same way than we did in Subsection 3.3 to compare output under both regimes, as they are continuous and monotonically decreasing over the relevant interval.

Defining the marketwide elasticity of demand, $\widehat{\varepsilon}(q) \equiv -P(q) / [qP'(q)]$, elasticity of the slope of the inverse demand $\widehat{E} \equiv -qP''(q)/P'(q)$, and the elasticity of the conduct parameter $\varepsilon_\theta(q) \equiv -\theta(q) / [q\theta'(q)]$, we can state the following result:

Proposition 2. *Under imperfect competition, when the government seeks to maximize tax revenue, the equilibrium quantity is larger under an ad valorem regime than under a unit tax regime if and only if* $\widehat{E} < 1 + 1/\widehat{\varepsilon} - 1/\varepsilon_\theta$ *at equilibrium quantities.*

Proposition 2 extends the previous results stated in Proposition 1. It shows that a simple rule, building on the elasticity and its slope, allows to identify whether ad valorem taxation leads to larger total surplus than unit taxation, or the opposite, in any framework of imperfect competition. However, in contrast to the Cournot case with homogeneous products, the threshold on the slope of the elasticity of demand to rank both tax regimes in terms of welfare also depends on the elasticity of the conduct parameter, ε_θ.

Note that, as an index of market competitiveness, the conduct parameter typically decreases with quantity. Therefore, ε_θ is generally positive,[11] and the condition on the convexity of total market demand for the unit tax regime to welfare-dominate the ad valorem one is relaxed, as compared to that in the homogeneous Cournot case. Indeed, under imperfect competition, even demand forms that are less convex than the constant-elasticity one can induce this welfare ranking of tax regimes, in the classic case where the conduct parameter slopes down.

By contrast, whenever the conduct parameter is constant, we have $\theta' = 0$ everywhere, and, therefore, it is the *sign* of the slope of the elasticity of demand which determines under which taxation regime output is the less distorted at equilibrium, as in Proposition 1. For instance, in the models of price or quantity competition with linearly differentiated products *à la* Singh and Vives (1984), θ is constant and the elasticity of demand decreases in quantity; therefore, the ad valorem regime is the welfare-maximizing one. Alternatively, in the representative consumer model with CES as studied by Anderson, de Palma, and Thisse (1992), the conduct parameter and the elasticity of demand are constant, so both tax regimes lead to the same equilibrium output when the government maximizes tax revenue. In addition, conjectural variation models, such as studied by Delipalla and Keen (1992), also display a constant conduct parameter.

Finally, the condition on demand curvature and the conduct parameter for the ad valorem regime to welfare-dominate the unit one under the Leviathan hypothesis is also equivalent to under-shifting of the ad valorem tax. This is easily demonstrated by following Anderson, de Palma, and Kreider (2001a), who derive the conditions for under- and over-shifting of an ad valorem tax when firms sell differentiated products, using the Chamberlinian notation of *dd* and *DD* curves.[12] Indeed, observing that, in a symmetric

[11] In the logit model, for instance, ε_θ is always positive.

[12] The *dd* curve corresponds to one firm's demand as a function of a change in its own price only, keeping other prices as fixed, and the *DD* curve to one firm's demand when all firms' prices move simultaneously.

equilibrium, $\widehat{\varepsilon}$ and $\widehat{E} + 1/\varepsilon_\theta$ are equivalent to the elasticity of the *DD* curve and to the ratio of the elasticity of the slope of the *dd* curve with respect to the common price to the elasticity of the *DD* curve, respectively, we can apply their result and state that there is under-shifting of the ad valorem tax if and only if $\widehat{E} + 1/\varepsilon_\theta < 1 + \widehat{\varepsilon}$.

5 Conclusion

We demonstrated that the ad valorem tax regime welfare-dominates the unit one under a Leviathan government which maximizes revenue if and only if the elasticity of demand increases in price, in a wide set of competition frameworks, and with mild restrictions on the demand side. When the elasticity of demand decreases in price, then a unit tax regime is preferable from a welfare point of view. Therefore, the choice of a tax regime could serve as a constitutional constraint for Leviathan governments (see Brennan and Buchanan (1978)); indeed, they would always prefer an ad valorem regime, but this regime may not be the one maximizing total surplus.

Finally, an interesting extension of this work would be to integrate long-run effects due to free entry. Indeed, in our analysis, the number of firms in the market is exogenously determined, and, at equilibrium, the two tax regimes would typically lead to different surplus sharing between the government and firms. Therefore, were firms able to freely enter the market by incurring a fixed cost in a first stage, the equilibrium number of firms would typically differ under both tax regimes, and we might observe insufficient or excessive entry, which could have ambiguous welfare effects. This extension would be particularly challenging, however, in the general framework used in this paper.

Appendices

A Proofs for the Cournot framework

A.1 Proof of Lemma 1

With the definitions of the elasticity of demand, $\varepsilon(Q) \equiv -P(Q) / [QP'(Q)]$, and of the slope of the elasticity of demand, $E(Q) \equiv -QP''/P'$, it is easy to check that $\varepsilon' = (-\varepsilon/Q)(1 + 1/\varepsilon - E)$, and to see that $U(Q)$ in Definition 1 corresponds to the left-hand side of equation (2). In addition, we find that $\lim_{Q \to \hat{Q}^-} U(Q) = 0$, with $\hat{Q} \in (0, \infty)$ defined by $\lim_{Q \to \hat{Q}^-} E(Q) + n\varepsilon(Q) = 2 + n$. This proves existence of $\hat{Q}$. The second-order conditions given by Assumptions 1 and 2 imply that $U(\cdot)$ is positive and continuously decreasing on $\left(0, \hat{Q}\right)$. This also proves uniqueness of $\hat{Q}$.

A.2 Proof of Lemma 2

With the definitions of the elasticity of demand, and the slope of the elasticity of demand, it is easy to check that $A(Q)$ in Definition 2 corresponds to the left-hand side of equation (5). In addition, we find that $\lim_{Q \to \tilde{Q}^-} A(Q) = 0$, with $\tilde{Q} \in (0, \infty)$ defined by $\lim_{Q \to \tilde{Q}^-} \varepsilon(Q) = 1$. Moreover, Assumption 1 is equivalent to $E < 2$ and implies that the quantity $\hat{Q}$ defined in the unit tax regime is such that $\lim_{Q \to \hat{Q}^-} \varepsilon(Q) \geq 1$. This proves existence of $\tilde{Q} \in [\hat{Q}, \overline{Q}]$. The second-order conditions given by Assumptions 1 and 3 imply that $A(\cdot)$ is positive and continuously decreasing on $\left(0, \tilde{Q}\right)$. This also proves uniqueness of $\tilde{Q}$.

A.3 Proof of Lemma 3

Regarding $sign\,\{A(Q_u) - U(Q_u)\}$, since $U(Q_u) = A(Q_a) = c$, we have that

$$A(Q_u) - U(Q_u) > 0 \Leftrightarrow A(Q_u) - c > 0 \Leftrightarrow A(Q_u) - A(Q_a) > 0.$$

We now show that $A(Q_u) - A(Q_a) > 0 \Leftrightarrow Q_a > Q_u$. As explained in the proof of Lemma 2, $\hat{Q} \leq \tilde{Q}$ so we have $Q_u \leq \tilde{Q}$. Since, of the same Lemma, $A(\cdot)$ is decreasing over the interval $\left(0, \tilde{Q}\right)$, this equivalence holds.

Regarding $sign\,\{A(Q_a) - U(Q_a)\}$, since, by Assumption 2, $U(\cdot)$ is decreasing, an argument holds analogous to that of the first case above, showing that $U(Q_u) - U(Q_a) > 0 \Leftrightarrow Q_a > Q_u$.

A.4 Proof of Lemma 4

We have that $A(Q) - U(Q) = MR(Q) * (a/(1-b)) - MR(Q) * (a+b)$, where $a \equiv 1 - 1/\varepsilon$ and $b \equiv \varepsilon' Q/(\varepsilon(n\varepsilon - 1))$. We thus need to show that

$$\frac{a}{1-b} - (a+b) > 0 \quad \Leftrightarrow \quad 1 + \frac{1}{\varepsilon} - E > 0. \tag{8}$$

We now establish that $b < 1$. As noted in equation (3), $\varepsilon' = (-\varepsilon/Q)(1 + 1/\varepsilon - E)$. Plugging this into b and manipulating gives that $b < 1 \Leftrightarrow n\varepsilon + 1/\varepsilon - E > 0$. Assumption 1 is equivalent to $E < 2$, and thus $n\varepsilon + 1/\varepsilon - E > n\varepsilon + 1/\varepsilon - 2$. Rewriting $n\varepsilon + 1/\varepsilon - 2$ as $(\varepsilon - 1/n)^2 n/\varepsilon + (1 - 1/n)/\varepsilon$ and observing that $MR > 0 \Leftrightarrow \varepsilon > 1/n$, we obtain $n\varepsilon + 1/\varepsilon - E > 0$.

Since $b < 1$, inequality (8) can be rewritten as $b(1 - (a+b)) < 0$. Assumption 1 implies that $MR' < 0$, and since $a + b$ can be written as $1 + QMR'/MR$, it follows that $a + b < 1$. Therefore, $sign\{A(Q) - U(Q)\} = sign\{-b\}$ and the equivalence stated in (8) is true.

B Illustrative examples

Figure 1 below presents the Laffer curves for both the unit and ad valorem regimes in a monopoly market, under three different demand scenarios. The linear demand (top left panel) satisfies increasing elasticity of demand in price everywhere, and, thus, the equilibrium quantity in the ad valorem regime is larger than that in the unit regime. With a constant-elasticity demand (top right panel), the derivative of the elasticity equals zero everywhere. Therefore, equilibrium quantities are exactly the same under both regimes. Finally, we use the Adjustable pass-through (Apt) demand defined by Fabinger and Weyl (2012), to illustrate the case of decreasing elasticity of demand in price (i.e., increasing elasticity of demand in quantity).[13] It is given by

$$D(p) = \sigma \left(\frac{1 + \mu(1+\alpha)\left(\sqrt{\frac{\frac{p}{m}+\alpha}{1+\alpha}} - 1\right)}{1 + \mu(1+\alpha)\left(\sqrt{\frac{\alpha}{1+\alpha}} - 1\right)} \right)^{\frac{-2}{\mu}}.$$

The Apt demand with parameters $m = 0.01$, $\sigma = 0.5$, $\alpha = 0.2$, and $\mu = 0.7$ displays an increasing elasticity of demand in quantity at low quantities, that is, everywhere in the case illustrated in the bottom panel of Figure 1. In this case, the unit tax regime leads to a larger

[13] Polynomial forms, which can be used as a demand proxy for many distributions, as explained by Fabinger and Weyl (2014), are also flexible enough to allow for the case of decreasing elasticity of demand in price. As an example, $P(Q) = 2.88 + 4.42Q^{-2/3} - 2.34Q^{2/3} + 3.66Q^{4/3} - 8.62Q^3$ gives $E > 1 + 1/\varepsilon$ for a wide range of quantities.

equilibrium quantity than the ad valorem one, and, therefore, to a larger total surplus.[14]

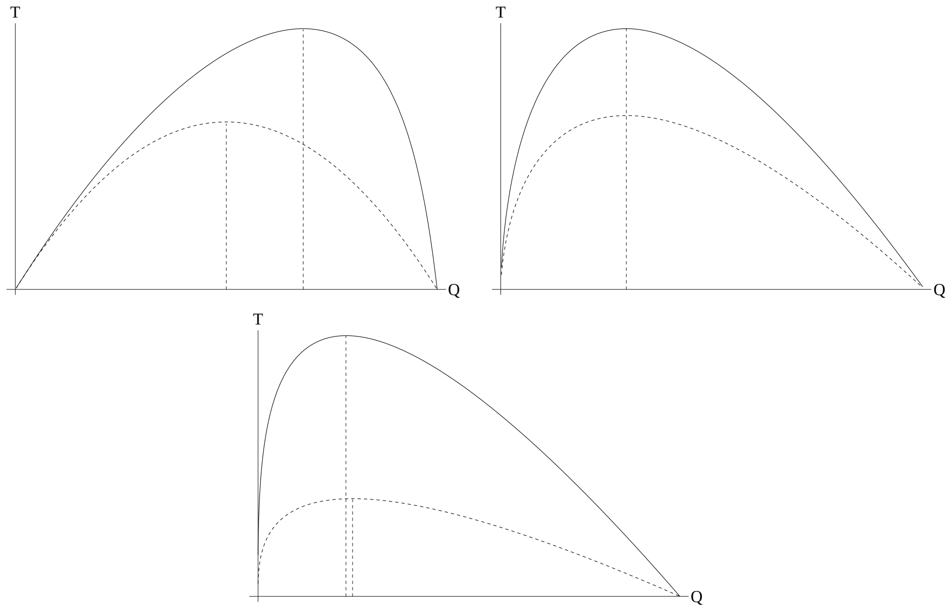

Figure 1: Tax revenue in unit (dashed) and ad valorem (plain) tax regimes as a function of the quantity set by a monopolist in response to a tax level/rate. Top Left: Linear demand, $D(p) = 1 - p$ with $c = 0.1$. Top Right: Constant-elasticity demand, $D(p) = p^{-3}$ with $c = 0.8$. Bottom: Adjustable pass-through demand from Fabinger and Weyl (2012) with parameters $m = 0.01$, $\sigma = 0.5$, $\alpha = 0.2$, $\mu = 0.7$, and with $c = 0.04$.

C General case with differentiated products

C.1 Definition

There are n firms, all with constant marginal costs c. Following Weyl and Fabinger (2013), we define firm i's conduct parameter, $\theta_i \in [0, 1]$, as

$$\theta_i \equiv \frac{(\mathbf{P} - \mathbf{c})\ \, d\mathbf{q}/d\sigma_i}{-\mathbf{q}\ d\mathbf{P}/d\sigma_i}, \tag{9}$$

where $\mathbf{q}$, $\mathbf{P}$, and $\mathbf{c}$ are vectors of firms' quantities, prices, and marginal costs, respectively, and where σ_i is firm i's strategic variable. This formulation leads to, in a symmetric

[14]Whereas the results for the linear and constant-elasticity demand forms are robust to any level of marginal cost such that demand is positive at equilibrium, we assume a marginal cost that is large enough in our example with the Apt demand, so that the elasticity of demand is decreasing in price (or increasing in quantity) in equilibrium at this marginal cost.

equilibrium, the FOC given by equation (6). More precisely, when firms compete in quantities, it gives

$$\theta_i = -\frac{P_i(\mathbf{q}) - c}{q_i \Sigma_j \, \partial P_j(\mathbf{q})/\partial q_i}. \tag{10}$$

By contrast, when firms compete in prices,

$$\theta_i = 1 + \frac{\Sigma_{j \neq i} \, \partial q_j(\mathbf{P})/\partial P_i}{\partial q_i(\mathbf{P})/\partial P_i} = 1 - \Delta_i, \tag{11}$$

where Δ_i is the aggregate diversion ratio, popular in antitrust analysis (see Katz and Shapiro (2003)). In addition, equation (6) can be rewritten as

$$P + \theta\left(q\left(P\right)\right)\frac{q\left(P\right)}{q'\left(P\right)} = c, \tag{12}$$

where P is the equilibrium price, and $q = \Sigma_i \, q_i(P)$ is total market demand (see Mahoney and Weyl (2014)). Therefore, $q'\left(P\right)$ is a measure of the marketwide margin, that is, consumers that are not served by the market anymore when all firms increase their prices simultaneously.

C.2 Equilibria

We assume that $P\left(\cdot\right)$ and $\theta\left(\cdot\right)$ are thrice and twice differentiable, respectively. We define a firm's marginal revenue as $\widehat{MR}\left(q\right) \equiv P\left(q\right) + q\theta\left(q\right)P'\left(q\right)$, with first and second derivatives $\widehat{MR}' = \left(1 + \theta + q\theta'\right)P' + q\theta P''$ and $\widehat{MR}'' = \left(2\theta' + q\theta''\right)P' + \left(1 + 2\theta + 2q\theta'\right)P'' + q\theta P'''$ (omitting arguments for clarity). This notation allows us to follow the equilibrium calculations for the Cournot model with homogeneous products from Section 3, where we had $q = Q$ and $\theta = 1/n$. As mentioned in the main text, we also define the following variables: the elasticity of (marketwide) demand, $\widehat{\varepsilon}(q) \equiv -P\left(q\right)/\left[qP'\left(q\right)\right]$, the elasticity of the slope of the inverse demand $\widehat{E} \equiv -qP''\left(q\right)/P'\left(q\right)$, and the elasticity of the conduct parameter $\varepsilon_\theta\left(q\right) \equiv -\theta\left(q\right)/\left[q\theta'\left(q\right)\right]$.

Following Section 3, the last-stage second-order condition (i.e., stability condition) is equivalent to $\widehat{MR}' < 0 \Leftrightarrow \widehat{E} + 1/\varepsilon_\theta < 1 + 1/\theta$, for a given θ, under unit and ad valorem taxation. Similar to Assumption 1, which was equivalent to $E < 2$, we assume that the stability condition holds for any $\theta \in [0, 1]$.

Assumption 4 (Stability condition)**.** $\widehat{E} + 1/\varepsilon_\theta < 2$.

Then, the first-order conditions for the government give, at equilibrium, the adjusted-marginal revenue curves $\widehat{U}\left(\cdot\right)$ and $\widehat{A}\left(\cdot\right)$ displayed in the equation system (7), in both unit and ad valorem tax regimes, respectively.

We assume the following second-order condition is satisfied in the unit tax regime.

Assumption 5 (Government's SOC - Unit tax regime)**.** *$\widehat{U}'(q) < 0$, $\forall q$ such that $\widehat{U}(q) > 0$.*

Similarly, for the ad valorem tax regime, we assume that the following second-order condition is satisfied.

Assumption 6 (Government's SOC - Ad valorem tax regime)**.** *$\widehat{A}'(q) < 0$, $\forall q$ such that $\widehat{A}(q) > 0$.*

Thanks to the second-order condition and our assumptions that $P(\cdot)$ and $\theta(\cdot)$ are differentiable, we are able to invert the adjusted marginal revenue curves $\widehat{U}(\cdot)$ and $\widehat{A}(\cdot)$, which are continuous and monotone over the relevant interval. This allows us to state that, for instance, the equilibrium output will be larger under the ad valorem regime for a given marginal cost if and only if the Ad valorem tax-adjusted marginal revenue curve lies above the Unit tax-adjusted one at the unit tax regime equilibrium quantity.

C.3 Proof of Proposition 2

The proof is similar to that obtained in the homogeneous Cournot case. Indeed, Assumptions 5 and 6 allow us to follow Lemma 3 and show that it is irrelevant at which equilibrium quantity we compare the two adjusted-marginal revenue curves. Besides, by using Assumption 4 we can follow Lemma 4 and show that:

$$sign\left\{\widehat{A}(q) - \widehat{U}(q)\right\} = sign\left\{1 + 1/\widehat{\varepsilon}(q) - \left[\widehat{E}(q) + 1/\varepsilon_\theta\right]\right\}, \tag{13}$$

by simply replacing E by $\widehat{E} + 1/\varepsilon_\theta$, ε by $\widehat{\varepsilon}$, and $1/n$ by θ, when deriving the result. Note that this result holds because, from the equilibrium equation (6), we always have $\widehat{\varepsilon} > \theta$, just as we had $\varepsilon > 1/n$ in the Cournot case.

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