Multi-Seller Access Pricing for Shopping Platforms: Online Appendix

Ming Gao*

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1 Demand Characterization with Bundles

In this section, we derive demand functions from the "product bundle" perspective. Although the functions are identical to those presented in the main text, this alternative method enables us to conduct more in-depth demand analysis which forms the basis of the additional results shown in this appendix, such as those on the multi-tier access prices in Section 3.

What combination, or bundle, of products each shopper purchases (if any) depends on his type $\mathbf{x}$ and the final prices he perceives for all possible bundles. Any bundle, denoted by J, is a subset of the full bundle N. The empty bundle is $\varnothing \subseteq N$.

A general profile of the platform's access price m and all sellers' prices $\mathbf{p} \equiv (p_1, ..., p_n)$ is called a *price schedule*, denoted $(m, \mathbf{p})$. Given $(m, \mathbf{p})$, the final price that a shopper who chooses bundle $J \subseteq N$ pays, denoted y_J, is given by

$$y_J = \begin{cases} \sum\limits_{j \in J} p_j + m & , \text{ if } J \neq \varnothing; \\ 0 & , \text{ if } J = \varnothing; \end{cases} \tag{1.1}$$

which essentially defines a *multiproduct two-part tariff.* The platform's access price m applies if you buy anything, whereas each seller's price applies only if you buy from that seller. A change in m will therefore change the final prices of *all* non-empty bundles by the same amount.

*School of Economics and Management, Tsinghua University, Beijing 100084, China. Email: gaom@sem.tsinghua.edu.cn.

Given $(m, \mathbf{p})$, shopper $\mathbf{x} =(x_1, ..., x_n)$ chooses bundle $J \neq \varnothing$, if and only if

$$\sum_{j\in J}(x_j - p_j) - m \geq \max\{0, \sum_{k\in K}(x_k - p_k) - m; \forall K \subseteq N, K \neq \varnothing\}, \tag{1.2}$$

which guarantees that J provides the highest surplus. Otherwise, he chooses the empty bundle. Denote $A_J(m, \mathbf{p})$ the set of all the shoppers who choose the non-empty bundle J, also called the demand segment of J, and we must have

$$A_J(m, \mathbf{p}) = \{\mathbf{x} \in I^n | \mathbf{x} \text{ satisfies (1.2)}\}. \tag{1.3}$$

Whenever indifferent, we assume that a shopper chooses the largest bundle, or randomizes with equal probabilities among equal-sized bundles.[1] For any $A \subseteq I^n$, we denote the probability measure of A as $\Pr[\mathbf{x} \in A] = \int_A f(\mathbf{x})d\mathbf{x}$.

Demand

Definition 1 (Individual Seller's Demand) *Given* $(m, \mathbf{p})$*, the demand segment of seller* j *is the set of all shoppers who buy bundles that contain product* j*, denoted*

$$\mathcal{B}_j(m, \mathbf{p}) \equiv \bigcup\nolimits_{J\ni j} A_J(m, \mathbf{p}), \tag{1.4}$$

and the demand for seller j *is the probability measure of* $\mathcal{B}_j(m, \mathbf{p})$*, denoted*

$$D_j(m, \mathbf{p}) \equiv \int_{\mathcal{B}_j(m,\mathbf{p})} f(\mathbf{x})d\mathbf{x}. \tag{1.5}$$

Note that $\mathcal{B}_j(m, \mathbf{p})$ represents all shoppers who buy product j, no matter if they also buy other products. Seller j's demand segment is denoted by calligraphic $\mathcal{B}_j$ for distinction from the demand segment of the single-product bundle $A_j(m, \mathbf{p})$. In fact, by (1.4), $\mathcal{B}_j(m, \mathbf{p}) \supseteq A_j(m, \mathbf{p})$.

Definition 2 (Platform's Demand) *Given* $(m, \mathbf{p})$*, denote the set of all shoppers who visit the platform (i.e. all purchasing shoppers)*

$$\mathcal{B}_0(m, \mathbf{p}) \equiv \bigcup\nolimits_{j\in N} \mathcal{B}_j(m, \mathbf{p}) = \bigcup\nolimits_{J\neq\varnothing} A_J(m, \mathbf{p}) = I^n \backslash A_\varnothing(m, \mathbf{p}), \tag{1.6}$$

[1] From (1.3) we know that all demand segments are *closed*. The intersection of different demand segments defines their "boundary", i.e. the set of indifferent shoppers. As density f has no atoms, shoppers on any boundary have zero mass and therefore do not pose a problem for demand measurement based on f to be defined shortly.

and the platform's demand is the probability measure of $\mathcal{B}_0(m, \mathbf{p})$, denoted

$$D_0(m, \mathbf{p}) \equiv \int_{\mathcal{B}_0(m,\mathbf{p})} f(\mathbf{x}) d\mathbf{x}. \tag{1.7}$$

Lemma 1.1 *Given $(m, \mathbf{p})$, for any $j \in N$, $D_j(m, \mathbf{p})$ and $D_0(m, \mathbf{p})$ are well defined and their expressions are given in the proof of Lemma 1 in the main text.*

For expository simplicity, all proofs are put into Section 5 of this document.

Given $(m, \mathbf{p})$, the maximized aggregate consumer surplus is given by equation (5) in the main text, and is repeated below for convenience:

$$V(m, \mathbf{p}) \equiv \mathbf{E}_{\mathbf{x}}[\max\{0, \sum_{k \in K}(x_k - p_k) - m, K \subseteq N, K \neq \varnothing\}], \tag{1.8}$$

Using the demand functions and $V(m, \mathbf{p})$, we have the next result.

Lemma 1.2 *When $n \geq 2$, given $(m, \mathbf{p})$, suppose there exist $j, k \in N$, such that $j \neq k$, $D_j > 0$, and $D_k > 0$. Then we have*

i) $\frac{\partial D_j}{\partial m} = \frac{\partial D_0}{\partial p_j} < 0$,

ii) $\frac{\partial D_0}{\partial m} < \frac{\partial D_j}{\partial m}$, *and*

iii) $\frac{\partial D_j}{\partial p_j} < \frac{\partial D_0}{\partial p_j}$.

In the following sections, whenever sharing between sellers and the platform is relevant, we only use *profit sharing* to illustrate the results and their intuition.

2 Properties of the Best-Response Functions

2.1 Sellers' best response

Seller j's best response to m and $\mathbf{p}_{-j}$ is defined by her first-order condition in the main text, as an implicit function $p_j^*(m, \mathbf{p}_{-j})$. It has the following property.

Proposition 2.1 (Seller's Best Response) *i)* $\frac{\partial p_j^*}{\partial m} < 0$ *if* $\frac{\partial^2 D_j}{\partial p_j^2} \leq 0$ *and* $\frac{\partial^2 D_0}{\partial p_j^2} \leq 0$*;*

ii) For $k \neq j$, $\frac{\partial p_j^*}{\partial p_k} < 0$ *if* $\frac{\partial D_j}{\partial p_k} < 0$, $\frac{\partial^2 D_j}{\partial p_j^2} \leq 0$ *and* $\frac{\partial^2 D_j}{\partial p_j \partial p_k} \leq 0$.

Part i) shows that a seller may respond to an increase in the platform's access price with a lower price under some conditions, which indicates that these prices are strategic substitutes. The conditions $\frac{\partial^2 D_j}{\partial p_j^2} \leq 0$ and $\frac{\partial^2 D_0}{\partial p_j^2} \leq 0$ require that the demand of seller j and that of the platform are both weakly concave in the seller's price. They hold, for instance, when f is the uniform distribution, where $\frac{\partial^2 D_j}{\partial p_j^2} = 0$ and $\frac{\partial^2 D_0}{\partial p_j^2} < 0$.

Part ii) shows that different sellers' prices are also strategic substitutes under some conditions. Condition $\frac{\partial D_j}{\partial p_k} < 0$ requires that j and k are complements, and this happens if and only if $m > 0$ according to Lemma 1. Condition $\frac{\partial^2 D_j}{\partial p_j \partial p_k} \leq 0$ requires that seller j's demand is submodular in its own price and another seller k's price, which also holds when f is the uniform distribution, where $\frac{\partial^2 D_j}{\partial p_j \partial p_k} = 0$.

2.2 The platform's best response

First-order condition (7) in the main text defines the platform's optimal strategy as an implicit function of sellers' prices $\mathbf{p}$ and of the profit shares it takes from sellers' profits, $m^*(\mathbf{p}, \{\beta_k\}_{k \in N})$. The next results come from a comparative-statics analysis.

Proposition 2.2 (Impact of Bargaining Power on Optimal Access Price) *Suppose $g_j'(\alpha) > 0$ for any $\alpha \in [0,1]$. Then more bargaining power of the platform relative to seller j leads to a lower access price, i.e. $\frac{\partial m^*}{\partial \alpha_j} < 0$, if and only if*

$$\sum_{k \in N} \beta_k \cdot \frac{\eta_k}{\epsilon_k} \left(\frac{\partial^2 D_0 / \partial m^2}{\partial D_0 / \partial m} - \frac{\partial^2 D_k / \partial m^2}{\partial D_k / \partial m} \right) < 2\sigma_0 + \frac{\partial^2 D_0 / \partial m^2}{\partial D_0 / \partial m}. \tag{2.1}$$

Under condition (2.1), Proposition 2.2 implies that the platform's bargaining power relative to each individual seller monotonically and negatively affects its equilibrium level of access price. The following result provides a simpler sufficient condition for $\frac{\partial m^*}{\partial \alpha_j} < 0$.

Corollary 2.3 *Condition (2.1) is implied by the following conditions*

$$\frac{\partial^2 D_0}{\partial m^2} \leq 0, \text{ and } \frac{\partial^2 D_0 / \partial m^2}{\partial D_0 / \partial m} < \frac{\partial^2 D_k / \partial m^2}{\partial D_k / \partial m} \text{ for any } k \in N. \tag{2.2}$$

Condition $\frac{\partial^2 D_0}{\partial m^2} \leq 0$ requires that the platform's demand is weakly concave in the access price. It holds, for instance, when f is the uniform distribution. Condition $\frac{\partial^2 D_0 / \partial m^2}{\partial D_0 / \partial m} < \frac{\partial^2 D_k / \partial m^2}{\partial D_k / \partial m}$ means that $\partial D_0 / \partial m$ is less elastic than $\partial D_k / \partial m$, when elasticities are measured with respect to m.

Proposition 2.4 (Platform's Best Response) $\frac{\partial m^*}{\partial p_j} < 0$ *if condition (2.1) holds, $m^* \geq c$, $\frac{\partial^2 D_j}{\partial m^2} \leq 0$ and $\frac{\partial^2 D_0}{\partial p_j \partial p_k} \leq 0$ for any $k \in N$.*

Condition $\frac{\partial^2 D_j}{\partial m^2} \leq 0$ requires that seller j's demand is weakly concave in the access price. Condition $\frac{\partial^2 D_0}{\partial p_j \partial p_k} \leq 0$ requires that platform's demand is submodular in j's and any other seller's prices. They hold, for instance, when f is the uniform distribution.

3 Multi-Tier Access Prices

In real life, multi-seller platforms often offer discounts when there are access fees. For instance, parking charges at a shopping mall may vary according to how much shoppers purchase at the mall. A seemingly popular practice is that a shopper gets a discount on parking fee once his total expenditure or total number of items purchased exceeds some threshold. This in effect creates two different tiers of access fees: shoppers with purchases lower than the threshold are charged a higher fee, say m, whereas shoppers with purchases higher than the threshold pay a lower fee, say $(m-e)$, with a discount $e > 0$.

In this section, we use our model to study when introducing such a discount would be profitable. Suppose the sellers charge $\mathbf{p}$, and the platform is considering whether to introduce a discount of $e > 0$ on its access price m, for shoppers who purchase $\underline{n}$ or more products. For any bundle $J \subseteq N$, denote $|J|$ its bundle size - the number of products in it.

Definition 3 (Two-Part Tariff with Access Discount) *A two-part tariff with an access discount for any bundle of at least $\underline{n}$ products, is a generalized price schedule*[2] $\mathbf{R} = \{r_J\}_{J \subseteq N}$ *consisting of four parts* $(m, \mathbf{p}, e, \underline{n})$*, where*

$$
\begin{aligned}
&(m, \mathbf{p}) \text{ is a two-part tariff defined in (1.1);} \\
&e \in (0, m]; \\
&r_J = \begin{cases} \sum_{j \in J} p_j + m - e & , \text{ if } |J| \geq \underline{n}; \\ \sum_{j \in J} p_j + m & , \text{ if } 0 < |J| < \underline{n}; \\ 0 & , \text{ if } J = \varnothing; \end{cases} \\
&\text{and } 2 \leq \underline{n} \leq n.
\end{aligned}
\tag{3.1}
$$

If $\underline{n} = 1$, it is clear that $\mathbf{R} = (m - e, \mathbf{p})$, exactly the same as a two-part tariff with access price $(m - e)$, and the discount e in is case has no different role than

[2] The "most general" price schedule in this model consists of 2^n prices, as there are 2^n possible bundles (including the empty bundle). Because sellers and the platform are not allowed to coordinate on prices, the relevant price schedules we need to consider are simpler.

the original access price m. Therefore, in this section we focus on the case when the threshold $\underline{n} \geq 2$.

Suppose the platform originally charges shoppers a fee, then offering a small discount e to all shoppers who buy at least $\underline{n}$ products results in *decreases* in demand for all bundles of less than $\underline{n}$ products and *increases* in demand for all bundles of at least $\underline{n}$ products. Note that the decrease in demand for bundles of less than $\underline{n}$ products is *not* due to shoppers leaving the platform. An additional discount on the access fee is no bad news to any shopper and therefore will not discourage entry. On the contrary, the decrease in demand for smaller bundles is due to marginal shoppers of smaller bundles now switching to larger bundles in order to qualify for the discount. (Figure 4 in the proof of Proposition 3.2 illustrates the demand changes induced by the access discount.)

Similar to (1.8), we re-define the maximized aggregate consumer surplus given $\mathbf{R} = (m, \mathbf{p}, e, \underline{n})$ in (3.1), denoted

$$V'(m, \mathbf{p}, e, \underline{n}) \equiv \mathbf{E}_{\mathbf{x}}[\max\{0, \sum_{k \in K, |K| \geq \underline{n}} (x_k - p_k) - m + e, \sum_{k \in K, |K| < \underline{n}} (x_k - p_k) - m, \forall K \subseteq N, K \neq \varnothing\}]. \tag{3.2}$$

and assume that V' is twice differentiable. Denote $D_{(|J| \geq \underline{n})}(\mathbf{R})$ the total demand for all multi-seller bundles of at least $\underline{n}$ products given $\mathbf{R}$, that is,

$$D_{(|J| \geq \underline{n})}(\mathbf{R}) \equiv \int_{\cup_{J \subseteq N, |J| \geq \underline{n}} A_J(\mathbf{R})} f(\mathbf{x}) d\mathbf{x}.$$

Using (3.2) and the demand functions defined previously in (1.5) and (1.7), by a similar envelope argument as before, we have, for any $j \in N$,

$$D_{(|J| \geq \underline{n})} = \frac{\partial V'}{\partial e}, \ D_j = -\frac{\partial V'}{\partial p_j}, \text{ and } D_0 = -\frac{\partial V'}{\partial m},$$

and therefore the next result follows from the Slutsky symmetry of $V'(\cdot)$.

Lemma 3.1 *Given* $\mathbf{R} = (m, \mathbf{p}, e, \underline{n})$ *in (3.1), for any* $j \in N$*,*

$$\frac{\partial D_j}{\partial e} = -\frac{\partial D_{(|J| \geq \underline{n})}}{\partial p_j} > 0, \ \textit{and} \ \frac{\partial D_0}{\partial e} = -\frac{\partial D_{(|J| \geq \underline{n})}}{\partial m} > 0. \tag{3.3}$$

The platform's profit given $\mathbf{R} = (m, \mathbf{p}, e, \underline{n})$ is:

$$\pi'(m, \mathbf{p}, e, \underline{n}) \equiv \sum_{j \in N} \beta_j (p_j - c_j) D_j(\mathbf{R}) + (m - c) D_0(\mathbf{R}) - e D_{(|J| \geq \underline{n})}(\mathbf{R}).$$

When on earth would the platform have an incentive to offer such an access discount? The answer depends on the sign of $\frac{\partial}{\partial e}\pi'(m^*, \{p_j^*\}_{j\in N}, e = 0, \underline{n})$. That is, starting from the equilibrium prices without discount $(m^*, \mathbf{p}^*)$, the platform will have an incentive to offer an access discount if this raises its profit.

Denote

$\eta_m^{\underline{n}} \equiv -\frac{\partial D_{(|J|\geq \underline{n})}}{\partial m} \cdot \frac{m}{D_{(|J|\geq \underline{n})}}$, the elasticity of demand for bundles of at least $\underline{n}$ products, with respect to m, and

$\eta_j^{\underline{n}} \equiv -\frac{\partial D_{(|J|\geq \underline{n})}}{\partial p_j} \cdot \frac{p_j}{D_{(|J|\geq \underline{n})}}$, the elasticity of demand for bundles of at least $\underline{n}$ products, with respect to p_j.

Add $*$ to their equilibrium values without discount, and the next result follows.

Proposition 3.2 (Incentive for Access Discount) *Suppose without access discount, the platform's optimal access fee or subsidy is m^* and the sellers' optimal prices are $\mathbf{p}^*$, then the platform has an incentive to offer a discount to all shoppers who buy from at least $\underline{n}(\geq 2)$ sellers if*

$$\underbrace{\frac{m^* - c}{m^*} \cdot \eta_m^{\underline{n}*}}_{\substack{\textit{gain from increased demand for platform} \\ \textit{as a fraction of loss in discount}}} + \sum_{j\in N} \underbrace{\beta_j \cdot \frac{\eta_j^{\underline{n}*}}{\epsilon_j^*}}_{\substack{\textit{gain from increased demand for seller } j \\ \textit{as a fraction of loss in discount}}} > 1. \tag{3.4}$$

The profitability of a one-unit access discount depends on the trade-off between an increased demand for all multi-seller bundles of $\underline{n}$ or more products, and the loss in discount paid out to all shoppers purchasing such bundles, equal to $D_{(|J|\geq \underline{n})}$. From Lemma 3.1 we know that the access discount actually increases demand for all sellers and for the platform, and that these marginal increases are equivalent to the marginal increases in the demand for bundles of $\underline{n}$ or more products, induced by a price cut in p_j and in m, respectively. The second term on the left-hand side of (3.4) represents the sum of the platform's gains due to the increase in individual seller's demand, as a fraction of the loss $D_{(|J|\geq \underline{n})}$. Similarly, the first term represents the gain from the platform's existing access price due to the increase in its demand, also as a fraction of the loss. The platform therefore has an incentive to offer an access discount when it gains more than it loses.

Intuitively, it is more likely for an access discount to be profitable if the demand for all multi-seller bundles of at least $\underline{n}$ products is *more elastic*, when measured by either m or p_j, such that an access discount induces more gains for the platform.

Moreover, from the platform's first-order condition without discount (8) in the main text we know that

$$\frac{m^* - c}{m^*} = (1 - \sum_{j \in N} \beta_j \cdot \frac{\eta_j^*}{\epsilon_j^*})/\eta_0^*,$$

and therefore it is also more likely for an access discount to be profitable if the demand for the platform as a whole is *less elastic*, when measured by either m or p_j (i.e. a lower η_0^* or η_j^*), such that the platform has a *higher* profit margin without the discount, and can therefore gain more from the *higher* demand induced by the discount.

Note that condition (3.4) is evaluated at the optimal two-part tariff set by the platform and the sellers, $(m^*, \mathbf{p}^*)$, just as in the optimal access price (9) in the main text. However, there is an interesting contrast between these conditions. The elasticities in (9) are calculated using the total demand for the platform, whereas those in (3.4) *exclude* the demand for all bundles of less than $\underline{n}$ products.

The fundamental reason why offering a discount that in effect creates a second (and lower) tier of access fee can be profitable is that it achieves better price discrimination, under "favorable" conditions like (3.4). Similar arguments should apply to using more tiers of access prices.

4 A More General Framework

The purpose of this section is to show that, two assumptions used in the main text can be relaxed to include more general cases, without affecting the main findings and their intuition in the article. These assumptions are: Each shopper has unit demand for each product, and has additive valuations for different products. Both were invoked to reduce the burden on characterizing shopper demand.

In this section, they are relaxed to allow for *multi-unit* demand and *non-additive* valuations. We further allow for *more diverse preferences* of different shoppers, parameterized by general multi-dimensional types. As long as each shopper can still choose an optimal "shopping basket" of various sellers' products (possibly with multiple units), we will be able to define the platform's and each seller's demand in a similar way as in the main text. Therefore the equations for equilibrium seller prices and equilibrium access price remain the same as before.

However, the demand functions now depend both on shoppers' utility function, and the distribution function of their types. In order to rule out "ill-behaved" such functions, we introduce a new assumption - A2 (to be presented shortly) - which can be interpreted as a strong version of the law of demand. It requires that each shopper who visits the platform has a weakly downward-sloping demand curve for every product, other things being equal. This new assumption allows us to simplify the formula for the equilibrium access price, and derive the same sufficient conditions for an equilibrium fee/subsidy, as those presented in the original paper.

4.1 Modelling framework

The settings regarding the sellers and the platform are the same as in the main text. The setting about shoppers' preferences are different here.

Following Armstrong (1996), suppose that there is a continuum of shoppers of mass 1, who have a variety of preferences over these products parameterized by the l-dimensional real-valued vector $\boldsymbol{\theta} \equiv (\theta_1, ...\theta_l)$. Suppose that a type-$\boldsymbol{\theta}$ shopper's utility when he chooses a **shopping basket** $\mathbf{q} \equiv (q_1, ..., q_n) \in \mathbb{R}^{+n}$ and makes a total payment t is given by

$$u(\boldsymbol{\theta}, \mathbf{q}) - t$$

where $q_j = 0, 1, 2, ...$ represents the number of units of product j purchased, and t includes all prices he pays to the platform and to the relevant seller(s), minus any subsidies he obtains. Assume $u(\boldsymbol{\theta}, \mathbf{0}) = u(\mathbf{0}, \mathbf{q}) = 0$, and u is continuous and increasing in all arguments.

Shoppers are heterogeneous such that $\boldsymbol{\theta}$ varies across them following probability distribution F with density f. Denote F_j the marginal distribution of θ_j, and f_j its marginal density.[3] These distributions are known to the sellers and the platform. The support of f is denoted Ω.

Assumption A1 *Ω is a weakly convex and bounded subset of $\mathbb{R}^{+l}$ with full dimension; f is atomless and $f(\boldsymbol{\theta}) > 0$ if and only if $\boldsymbol{\theta} \in \Omega$.*

The assumption on the timing of the pricing game is the same as in the main text.

Demand Shoppers' demand depends on their type and the final prices they perceive for different shopping baskets, which include the prices charged (and subsidies

[3] When there is little risk of confusion, the same notations are used in this section for some variables or functions as those for their counterparts in the main text, although there may be differences in their definitions. This small abuse of notations can help us draw comparisons between the original model and the current more general one.

offered, if any) by different sellers and the platform. A profile of all sellers' prices is denoted (as a column vector) $\mathbf{p} \equiv (p_1, p_2, ..., p_n)^T \in \mathbb{R}^n$. Without any access fee or subsidy, the final price for shopping basket $\mathbf{q}$ would simply be $\mathbf{qp} = \sum_{j \in N} q_j p_j$.

When the platform sets an access price m that applies to all purchasing shoppers, it changes the final prices of *all* shopping baskets by the same amount m. Combined with the seller's prices $\mathbf{p}$, the resulting price schedule is essentially the following multiproduct two-part tariff.

Definition 4 (Two-Part Tariff for Shopping Basket) *A two-part tariff consists of two parts $(m, \mathbf{p})$, where $m \in \mathbb{R}$ is the platform's access price, $\mathbf{p} = (p_1, p_2, ..., p_n)^T$ is a profile of all sellers' prices, and the total payment for shopping basket $\mathbf{q}$ =$(q_1, ..., q_n)$ under $(m, \mathbf{p})$ is given by*

$$t(\mathbf{q}, m, \mathbf{p}) = \begin{cases} \mathbf{qp} + m = \sum_{j \in N} q_j p_j + m & , \text{ if } \mathbf{q} > \mathbf{0}. \\ 0 & , \text{ if } \mathbf{q} = \mathbf{0}. \end{cases} \tag{4.1}$$

Note that $\mathbf{q} > \mathbf{0}$ requires that $\mathbf{q}$ has at least one strictly positive element. Given a two-part tariff $(m, \mathbf{p})$ and its payment schedule t in (4.1), assume each shopper has an optimal shopping basket, denoted

$$\mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p}) \equiv \arg\max_{\mathbf{q}} [u(\boldsymbol{\theta}, \mathbf{q}) - t(\mathbf{q}, m, \mathbf{p})], \tag{4.2}$$

in which the optimal quantity purchased from seller j is denoted $q_j^*(\boldsymbol{\theta}, m, \mathbf{p})$. The maximized aggregate consumer surplus is denoted

$$V(m, \mathbf{p}) \equiv \mathbf{E}_{\boldsymbol{\theta}}[u(\boldsymbol{\theta}, \mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p})) - t(\mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p}), m, \mathbf{p})], \tag{4.3}$$

and assumed to be twice differentiable.

Given two-part tariff $(m, \mathbf{p})$, the demand for seller $j \in N$ is denoted

$$D_j(m, \mathbf{p}) \equiv \int_{\Omega} q_j^*(\boldsymbol{\theta}, m, \mathbf{p}) dF(\boldsymbol{\theta}) = \mathbf{E}_{\boldsymbol{\theta}}[q_j^*(\boldsymbol{\theta}, m, \mathbf{p})], \tag{4.4}$$

and the platform's demand is denoted

$$D_0(m, \mathbf{p}) \equiv \Pr[\mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p}) > \mathbf{0}] = 1 - \Pr[\mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p}) = \mathbf{0}]. \tag{4.5}$$

Using the demand functions and the aggregate consumer surplus, by an envelope argument we have, for any $j \in N$,

$$D_j = -\frac{\partial V}{\partial p_j}, \text{ and } D_0 = -\frac{\partial V}{\partial m}.$$

Therefore, similar to Lemma 1 in the main text, the next result follows immediately from the Slutsky symmetry of V in (4.3).

Lemma 4.1 *Given two-part tariff $(m, \mathbf{p})$, for any $j \in N$ such that $D_j > 0$, we have*

$$\frac{\partial D_j}{\partial m} = \frac{\partial D_0}{\partial p_j}.$$

4.2 Optimal pricing

Given the previous new general modelling framework, all discussions in Section 3 of the main text hold. In particular, the optimal pricing formulas for sellers and for the platform, presented in Lemma 2 and Proposition 1, all still hold in the current new context. So does the necessary and sufficient condition (10) for an equilibrium access subsidy in Proposition 2. We repeat these results here for further discussion.

Lemma 4.2 *Given m and $\mathbf{p}_{-j}$, seller j's optimal price p_j^* satisfies*

$$\frac{p_j^* - c_j}{p_j^*} = \frac{1}{\epsilon_j}, \text{ or equivalently, } p_j^* = c_j + \frac{1}{\sigma_j}. \tag{4.6}$$

Proposition 4.3 (Optimal Access Pricing) *Given sellers' equilibrium prices $\mathbf{p}^*$ and price elasticities of demand $\{\epsilon_j^*\}_{j\in N}$, the platform's equilibrium access price m^* is given by*

$$m^* = c + \frac{1}{\sigma_0^*}(1 - \sum_{j\in N} \beta_j \cdot \frac{\eta_j^*}{\epsilon_j^*}). \tag{4.7}$$

Proposition 4.4 (Choice between Access Fee and Subsidy) *In equilibrium, the platform offers an access subsidy (i.e. $m^* < 0$) if and only if*

$$\sum_{j\in N} \beta_j \cdot \frac{\eta_j^*}{\epsilon_j^*} > 1 + c\sigma_0^*. \tag{4.8}$$

In order to further derive sufficient conditions for an equilibrium fee and a subsidy, we need the following assumption.

Assumption A2 (Law of Demand) $\frac{q_j^*(\boldsymbol{\theta}, m, \mathbf{p})}{\Pr[\mathbf{q}^*(\boldsymbol{\theta}, m, \mathbf{p}) > 0]}$ *is non-increasing in p_j, for any $\boldsymbol{\theta} \in \Omega$ and $j \in N$.*

Assumption A2 represents a strong version of the "law of demand", which requires that each shopper who enters the platform has a (weakly) downward-sloping demand curve for every product, other things being equal. Because q_j^* and $\Pr[\mathbf{q}^* > 0]$ depend on both the utility function u and the distribution function F, Assumption A2 rules out other "ill-behaved" such functions. The model in our original paper is clearly a special case where this assumption holds.

We call Assumption A2 a "strong" version for two reasons: 1) it applies to a shopper's demand for j *conditional on* him staying with the platform; and 2) it applies to *individual* demand rather than aggregate demand. In fact, Assumption A2 implies the following similar conclusion as Lemma 3 in the main text, which will help us simplify condition (4.8).

Lemma 4.5 *Under Assumption A2, when $n \geq 2$, the platform's demand is less elastic than each seller's demand when elasticities are measured by that seller's price. That is, for any $j \in N$, such that $D_j > 0$, we have*

$$\eta_j \leq \epsilon_j.$$

Therefore $\sum_{j \in N} \beta_j \frac{\eta_j^*}{\epsilon_j^*} \leq \sum_{j \in N} \beta_j$, and the next results follow immediately from (4.8).

Corollary 4.6 (Equilibrium Access Fee) *Under Assumption A2, the platform charges an access fee ($m^* \geq c \geq 0$) in equilibrium if*

$$\sum_{j \in N} \beta_j \leq 1. \tag{4.9}$$

Corollary 4.7 (Equilibrium Access Subsidy) *Under Assumption A2, the platform offers an access subsidy ($m^* < 0$) in equilibrium if*

$$\sum_{j \in N} \beta_j > \frac{1}{\hat{\lambda}}(1 + c\sigma_0^*). \tag{4.10}$$

Note that Corollary 3 in the main text does not depend on Assumption A2.

4.3 Model in original paper as a special case

The model in our original paper can be seen as the following special case: Each shopper demands 0 or 1 unit of each product, i.e., $q_j \in \{0, 1\}$; $l = n$ such that the taste parameter $\boldsymbol{\theta}$ is now an n-dimensional real-valued vector, and $\theta_j = x_j$, which represents a shopper's valuation of (or the utility he derives from) product j; these valuations are *additive* such that the net utility a shopper derives from purchasing a set of products simply equals the sum of his valuations for these products, minus all the prices he pays.

When shopper $\boldsymbol{\theta}$ (or equivalently $\mathbf{x}$) purchases from any non-empty subset $J \subseteq N$ of sellers, given the two-part tariff $(m, \mathbf{p})$ in (4.1), his utility is

$$\sum_{j \in J} (x_j - p_j) - m.$$

All the results in the original paper therefore follow directly.

5 Proofs

In these proofs, it is sometimes more convenient to use *general price schedules*, denoted $\mathbf{P} \equiv \{p_J\}_{J \subset N}$, where $p_J \in \mathbb{R}$ for any $J \subset N$. We assume $p_\varnothing = 0$ in all price schedules without loss of generality.

Proof of Lemma 1.1 Given $(m, \mathbf{p})$, we provide and prove the following three lemmas (5.1 through 5.3) which characterize the allocations of shoppers, and then $D_0(m, \mathbf{p})$ and $D_j(m, \mathbf{p})$ follow by definition. For expository simplicity, in the following characterization we only consider price schedules where $p_j \geq 0$ for all $j \in N$. A negative price proves unprofitable in seller j's maximization problem.

The bundle that only contains product j is denoted $\{j\}$, and simplified to j when it does not cause confusion. Denote $J^C \equiv N \backslash J$ the complementary bundle of J. j^C simply means $\{j\}^C$. For any bundle $J \subset N$, denote $|J|$ its bundle size - the number of products in it. A shopper *allocation* given price schedule $\mathbf{P}$ is the profile of demand segments of all bundles induced by $\mathbf{P}$, denoted $\{A_J(\mathbf{P})\}_{J \subset N}$. A price schedule $\mathbf{P} \equiv \{p_J\}_{J \subset N}$ is called *additive* if $p_\varnothing = 0$ and $p_J = \sum\limits_{j \in J} p_j$ for any non-empty $J \subset N$. An additive price schedule can also be simply written as $\mathbf{p} = (p_1, p_2, ..., p_n)$. From (1.1), we know the two-part tariff $(m, \mathbf{p})$ is additive if and only if $m = 0$. In the following proofs we often use $\mathbf{Y} \equiv \{y_J\}_{J \subset N}$ to represent the two-part tariff $(m, \mathbf{p})$ in (1.1) as a price schedule.

Lemma 5.1 (Additive Allocation) *If $\mathbf{P} = \{p_J\}_{J \subset N}$ is additive, the allocation it induces $\{A_J(\mathbf{P})\}_{J \subset N}$ satisfies for any $J \subset N$,*

$$A_J(\mathbf{P}) = \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C\} \tag{5.1}$$

As illustrated in Figure 1, an additive price schedule allocates all shoppers into "hyperrectangles" delineated by orthogonal hyperplanes, each defined by an equation $x_j = p_j$.

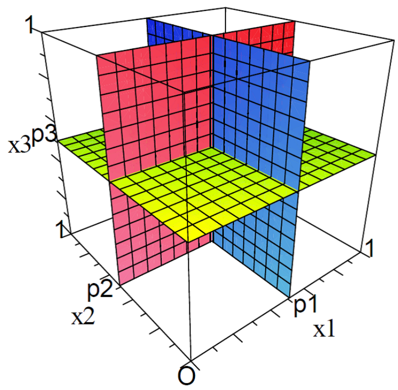
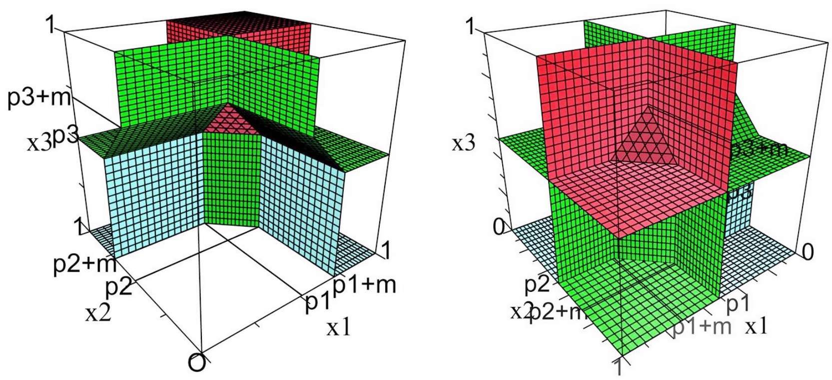


Figure 1. Additive Allocation ($n = 3, p_j = 0.5$)

Lemma 5.2 (Allocation under Access Fee) *When $m \geq 0$, the allocation $\{A_J(m, \mathbf{p})\}_{J \subset N}$ induced by $(m, \mathbf{p})$ in (1.1) satisfies, for any $J \subset N$,*

$$A_J(m, \mathbf{p}) = \begin{cases} \{\mathbf{x} \in I^n | \sum_{j \in J} x_j \geq \sum_{j \in J} p_j + m; x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C\} & , \textit{ if } J \neq \varnothing; \\ \{\mathbf{x} \in I^n | \sum_{k \in K} x_k < \sum_{k \in K} p_k + m, \forall K \neq \varnothing, K \subset N\} & , \textit{ if } J = \varnothing. \end{cases} \tag{5.2}$$



(On the Left: View from Origin) (On the Right: View Facing Origin)

Figure 2. Allocation under Access Fee ($n = 3, m = 0.2, p_j = 0.5$)

Lemma 5.3 (Allocation under Access Subsidy) *When $m \leq 0$, the allocation*

$\{A_J(m,\mathbf{p})\}_{J\subset N}$ *induced by* $(m,\mathbf{p})$ *in (1.1) satisfies, for any* $J \subset N$*,*

$$A_J(m,\mathbf{p}) = \begin{cases} \{\mathbf{x}\in I^n | x_j \geq p_j, \forall j \in J; x_k < p_k, \forall k \in J^C\} = A_J & , \text{ if } |J| > 1; \\ \{\mathbf{x}\in I^n | x_j - p_j \geq \max[m, x_k - p_k], x_k \leq p_k, \forall k \in j^C\} & , \text{ if } J = \{j\}; \\ \{\mathbf{x}\in I^n | x_k \leq p_k + m, \forall k \in N\} & , \text{ if } J = \varnothing. \end{cases} \tag{5.3}$$

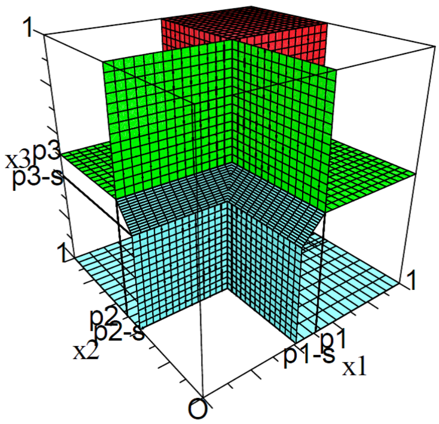


Figure 3. Allocation under Access Subsidy ($n = 3, s \equiv -m = 0.1, p_j = 0.5$)

Therefore $D_j(m,\mathbf{p})$ and $D_0(m,\mathbf{p})$ exist by Definitions 1 and 2. Now we prove these three lemmas on allocations.

Lemma 5.1 proof When $\mathbf{P} = \{p_J\}_{J\subset N}$ is additive, by definition the following two conditions are clearly equivalent for any $J \subset N$: (i) $\sum\limits_{j\in J} x_j - p_J \geq \sum\limits_{k\in K} x_k - p_K, \forall K \subset N$;(ii) $x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C$.

Lemma 5.2 proof For $(m,\mathbf{p})$ in (1.1), denote $\mathbf{P}$ =$(p_1, ..., p_n)$, which is the additive price schedule comprised only of the prices set by sellers. By Lemma 5.1 and (1.3), we know $A_J(\mathbf{P}) = \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k < p_k, \forall k \in J^C\}$.

By (1.1) and (1.3), $A_\varnothing(m,\mathbf{p}) = \{\mathbf{x} \in I^n | \sum\limits_{k\in K} x_k < \sum\limits_{k\in K} p_k + m, \forall K \neq \varnothing, K \subset N\}$; and $A_{J(\neq\varnothing)}(m,\mathbf{p}) = \{\mathbf{x} \in I^n | \sum\limits_{j\in J} x_j \geq \sum\limits_{j\in J} p_j + m; \sum\limits_{j\in J} x_j - \sum\limits_{j\in J} p_j \geq \sum\limits_{k\in K} x_k - p_K, \forall K \subset N\} = \{\mathbf{x} \in I^n | \sum\limits_{j\in J} x_j \geq \sum\limits_{j\in J} p_j + m; x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C\}$. The last equation is due to the equivalence between (i) and (ii) in the proof of Lemma 5.1.

Lemma 5.3 proof In order to distinguish access subsidies from fees, we denote a subsidy by $s \equiv -m$. In this proof, we require that $s \geq 0$. Use $\mathbf{Y}$ =$\{y_J\}_{J\subset N}$ to represent the two-part tariff $(m,\mathbf{p})$ in (1.1) as a price schedule, and we have $y_J = p_J - s$, if $J \neq \varnothing$; $y_J = 0$, if $J = \varnothing$. The allocation when $m = 0$ is defined in (5.1), and for any $J \subset N$, $A_J(\mathbf{Y})$ is given by (1.3). Now we study J of different sizes.

i) When $|J| > 1$, for any such $J \subset N$, we have $y_J = p_J - s$, which implies

$$\sum_{j \in J} x_j - y_J \geq \sum_{k \in K} x_k - y_K, \forall K \subset N \Leftrightarrow \begin{cases} \text{(a) } \sum_{j \in J} x_j - p_J \geq \sum_{k \in K} x_k - p_K & , \forall K \subset N, K \neq \varnothing; \\ \text{(b) } \sum_{j \in J} x_j - p_J \geq -s & , \text{ when } K = \varnothing. \end{cases}$$

From the equivalence between (i) and (ii) in the proof of Lemma 5.1, we know that for $|J| > 1$,

$$\text{part (a) above} \Leftrightarrow \begin{cases} \text{(a1) } x_j \geq p_j & , \forall j \in J; \\ \text{(a2) } x_k \leq p_k & , \forall k \in J^C. \end{cases}$$

And it is straightforward to see that (a1) implies (b) above, as $\mathbf{P} = (p_1, ..., p_n)$ is additive. This means that when we use an access subsidy $s > 0$, the participation condition of any multi-seller shopper is not binding.

From (a1) and (a2) we know, for any J such that $|J| > 1$, the two-part tariff with a subsidy $\mathbf{Y} = (m \leq 0, \mathbf{p})$ induces the *same* demand segment as $\mathbf{P}$:

$$A_{J,|J|>1}(\mathbf{Y}) = \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C\} = A_{J,|J|>1}(\mathbf{P})$$

ii) When $|J| = 1$, i.e. $J = \{j\}$. For any $j \in N$, $y_j = p_j - s$, which implies

$$\sum_{j \in J} x_j - y_J \geq \sum_{k \in K} x_k - y_K, \forall K \subset N \Leftrightarrow \begin{cases} \text{(c) } x_j - p_j \geq \sum_{k \in K} x_k - p_K & , \forall K \ni j, K \subset N; \\ \text{(d) } x_j - p_j \geq \sum_{k \in K} x_k - p_K & , \forall K \not\ni j, |K| > 1, K \subset N; \\ \text{(e) } x_j - p_j \geq x_k - p_k & , \forall k \in j^C; \\ \text{(f) } x_j - p_j \geq -s & , \text{ when } K = \varnothing. \end{cases}$$

If we write out the right-hand side of (c) above for K of different sizes, we have

$$\text{part (c) above} \Leftrightarrow \text{(c1) } x_k \leq p_k, \forall k \in j^C.$$

Also, it is clear that (c1) and (e) together imply (d) as $\mathbf{P}$ is additive. Therefore we only need (c1), (e) and (f) to fully characterize the demand segment of any single product $j \in N$ that $\mathbf{Y}$ induces, which is

$$A_j(\mathbf{Y}) = \{\mathbf{x} \in I^n | x_j - p_j \geq \max[-s, x_k - p_k], \text{ and } x_k \leq p_k, \forall k \in j^C\}$$

Note that this is different from the single-seller segments induced by $\mathbf{P}$ in (5.2).

iii) When $|J| = 0$, i.e. $J = \varnothing$, we have $y_\varnothing = p_\varnothing = 0$, which implies

$$\sum_{j\in J} x_j - y_J \geq \sum_{k\in K} x_k - y_K, \forall K \subset N \Leftrightarrow \begin{cases} \text{(g) } 0 \geq \sum\limits_{k\in K} x_k - p_K + s & , \forall K \subset N, |K| > 1; \\ \text{(h) } 0 \geq x_k - p_k + s & , \forall k \in N. \end{cases}$$

Clearly, (h) implies (g) as $s \geq 0$ and $\mathbf{P}$ is additive. Therefore we have

$$A_\varnothing(\mathbf{Y}) = \{\mathbf{x} \in I^n | x_k \leq p_k - s, \forall k \in N\}.\blacksquare$$

Proof of Lemma 1.2 For any $A \subset I^n$, denote the probability measure of A as $M(A) \equiv \Pr[\mathbf{x} \in A] = \int_A f(\mathbf{x})d\mathbf{x}$. Let $\mathbf{Y} = \{y_J\}_{J\subset N}$ be the price schedule representation of $(m, \mathbf{p})$ in (1.1). By Slutsky symmetry of V defined in (1.8), we have $\frac{\partial D_j}{\partial m} = -\frac{\partial^2 V}{\partial p_j \partial m} = \frac{\partial D_0}{\partial p_j}$.

Step 1: When $m \geq 0$, from (5.2) we observe that:

i) For any $J \subset N$, when m increases, $p_J + m$ also increases, therefore the set $A_J(\mathbf{Y})$ shrinks, i.e. $\frac{\partial M(A_J(\mathbf{Y}))}{\partial m} < 0$. Because $D_j = \sum_{J\ni j} M(A_J(\mathbf{Y}))$, we have $\frac{\partial D_j}{\partial m} = \sum_{J\ni j} \frac{\partial M(A_J(\mathbf{Y}))}{\partial m} < 0$, implying $\frac{\partial D_0}{\partial p_j} < 0$.

ii) Because $D_0 = \sum_{J\neq\varnothing} M(A_J(\mathbf{Y})) = D_j + \sum_{J\not\ni j} M(A_J(\mathbf{Y}))$, we have $\frac{\partial D_0}{\partial m} = \frac{\partial D_j}{\partial m} + \sum_{J\not\ni j} \frac{\partial M(A_J(\mathbf{Y}))}{\partial m} < \frac{\partial D_j}{\partial m} = \frac{\partial D_0}{\partial p_j}$.

iii) For any $J \not\ni j$, when p_j increases, the set $A_J(\mathbf{Y})$ expands, and therefore $\frac{\partial M(A_J(\mathbf{Y}))}{\partial p_j} > 0$. Because $D_0 = D_j + \sum_{J\not\ni j} M(A_J(\mathbf{Y}))$, we have $\frac{\partial D_j}{\partial p_j} = \frac{\partial D_0}{\partial p_j} - \sum_{J\not\ni j} \frac{\partial M(A_J(\mathbf{Y}))}{\partial p_j} < \frac{\partial D_0}{\partial p_j} = \frac{\partial D_j}{\partial m}$.

Step 2: When $m \leq 0$, from (5.3) we observe that:

i) $m = -s$ only affects $A_j(\mathbf{Y})$ for any $j \in N$. When $m(< 0)$ increases, $s(= -m > 0)$ decreases, and set $A_j(\mathbf{Y})$ shrinks, i.e. $\frac{\partial M(A_j(\mathbf{Y}))}{\partial m} < 0$. Because $\frac{\partial D_j}{\partial m} = \sum_{J\ni j} \frac{\partial M(A_J(\mathbf{Y}))}{\partial m} = \frac{\partial M(A_j(\mathbf{Y}))}{\partial m} < 0$.

ii) $\frac{\partial D_0}{\partial m} = \sum_{J\neq\varnothing} \frac{\partial M(A_J(\mathbf{Y}))}{\partial m} = \sum_{j\in J} \frac{\partial M(A_j(\mathbf{Y}))}{\partial m} < \frac{\partial D_j}{\partial m}$.

iii) For seller j, when p_j increases, every $A_J(\mathbf{Y})$ where $J \ni j$ shrinks, and therefore $\frac{\partial D_j}{\partial p_j} = \sum_{J\ni j} \frac{\partial M(A_J(\mathbf{Y}))}{\partial p_j} = \frac{\partial M(A_j(\mathbf{Y}))}{\partial p_j} + \sum_{J\ni j,|J|>1} \frac{\partial M(A_J(\mathbf{Y}))}{\partial p_j} < \frac{\partial M(A_j(\mathbf{Y}))}{\partial p_j}$. Now focus on $A_j(\mathbf{Y})$ in (5.3) and we observe that $x_j \geq p_j + \max[-s, x_k - p_k], \forall k \in j^C$. Therefore $\frac{\partial M(A_j(\mathbf{Y}))}{\partial p_j} \leq \frac{\partial M(A_j(\mathbf{Y}))}{\partial m} = \frac{\partial D_j}{\partial m}$, implying $\frac{\partial D_j}{\partial p_j} < \frac{\partial D_j}{\partial m}$.$\blacksquare$

Alternative Proof of Lemma 1 - Parts (ii) and (iii) - in the Main Text Using "Bundle Demand" **ii)** When $m > 0$, for any $J \ni j$, all the possible situations when the choice of a shopper $\mathbf{x} \in A_J(\mathbf{Y})$ can be affected as p_k increases by one unit:

a) $J \ni k$ but after p_k increases, $\mathbf{x}$ switches to $\varnothing$;

b) $J \ni k$ but after p_k increases, $\mathbf{x}$ switches to a bundle $J' \not\ni k$;

c) $J \ni k$ but after p_k increases, $\mathbf{x}$ switches to another bundle $J' \ni k$;

d) $J \not\ni k$ but after p_k increases, $\mathbf{x}$ switches to another bundle $J' \not\ni k$;

e) $J \not\ni k$ but after p_k increases, $\mathbf{x}$ switches to a bundle $J' \ni k$;

f) $J = \varnothing$ but after p_k increases, $\mathbf{x}$ switches to a bundle $J' \neq \varnothing$.

Because all other prices remain unchanged, e) and f) clearly are impossible as results of a price increase. d) is also impossible as no incentive compatible constraints between any two bundles that do not contain k are changed. For the same reason with respect to two bundles that both contain k, c) is also impossible.

J' in b) must be the bundle that excludes only k from J, i.e. $J' = J \backslash \{k\}$, because an increase in p_k only affects the incentive compatibility constraint between these two bundles, when one of them is J. Therefore, even though b) is possible, it does not affect D_j as both J and J' contain j.

Therefore only switching by shoppers of the kind in a) may induce some change in D_j. From (5.2) we observe that a) only occurs when the following participation constraint is binding for a bundle J that contains both j and k, and an increase in p_k tightens this constraint.

$$\sum_{l \in J} x_l \geq p_J + m = \sum_{l \in J} p_l + m = p_k + m + \sum_{l \in J, l \neq k} p_l$$

Therefore demand segments of such bundles (that contain both j and k) all shrink when p_k increases. In fact, we have

$\frac{\partial D_j}{\partial p_k} = \sum_{J \ni j,k} \frac{\partial M(A_J(\mathbf{Y}))}{\partial p_k} < 0.$

Note that, besides a) through f), the only other possibility for D_j to change as p_k does is:

g) When $\mathbf{x} \in A_J(\mathbf{Y})$ where $J \not\ni j$, $J \ni k$ but after p_k increases, $\mathbf{x}$ switches to a bundle $J' \ni j$, $J' \not\ni k$. However, this would require that the IC constraint between J and J' be affected by p_k, and there exists no such bundles in (5.2).

iii) When $m < 0$, we also have the possibilities from a) through g) as in ii). From (5.3), we observe that only g) is now a valid possibility, and the switching happens between single-seller bundles $\{k\}$ and $\{j\}$. An increase in p_k relaxes the following IC constraint between them, in favor of bundle $\{j\}$: $x_j - p_j \geq \max[m, x_k - p_k]$. Therefore $\frac{\partial D_j}{\partial p_k} = \frac{\partial M(A_j(\mathbf{Y}))}{\partial p_k} > 0.$■

Proof of Proposition 2.1 In (7) in the main text, let $p_j^* = p_j^*(m, \mathbf{p}_{-j})$ and take partial derivative with respect to m on both sides, we have

$$\frac{\partial D_j}{\partial m} + \frac{\partial D_j}{\partial p_j}\frac{\partial p_j^*}{\partial m} + \frac{\partial D_j}{\partial p_j}\frac{\partial p_j^*}{\partial m} + (p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j^2}\frac{\partial p_j^*}{\partial m} + (p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j \partial m} = 0.$$

$$\frac{\partial p_j^*}{\partial m} = -\frac{(p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j \partial m} + \frac{\partial D_j}{\partial m}}{2 \times \frac{\partial D_j}{\partial p_j} + (p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j^2}}.$$

By Lemma 1.2 we know $\frac{\partial D_j}{\partial m} < 0$, $\frac{\partial D_j}{\partial p_j} < 0$. By Slutsky symmetry of V defined in (1.8), we have $\frac{\partial^2 D_j}{\partial p_j \partial m} = \frac{\partial^2 D_0}{\partial p_j^2}$. Therefore $\frac{\partial p_j^*}{\partial m} < 0$ when $\frac{\partial^2 D_j}{\partial p_j^2} \leq 0$ and $\frac{\partial^2 D_0}{\partial p_j^2} \leq 0$.

For $k \neq j$, take partial derivative with respect to p_k on both sides of (7), we have

$$\frac{\partial p_j^*}{\partial p_k} = -\frac{(p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j \partial p_k} + \frac{\partial D_j}{\partial p_k}}{2 \times \frac{\partial D_j}{\partial p_j} + (p_j^* - c_j)\frac{\partial^2 D_j}{\partial p_j^2}}.$$

Therefore $\frac{\partial p_j^*}{\partial p_k} < 0$ when $\frac{\partial D_j}{\partial p_k} < 0$, $\frac{\partial^2 D_j}{\partial p_j^2} \leq 0$ and $\frac{\partial^2 D_j}{\partial p_j \partial p_k} \leq 0$.■

Proof of Proposition 2.2 Condition (10) in the main text defines $m^*(\{p_k\}_{k\in N}, \{\beta_k\}_{k\in N})$. Take partial derivative with respect to β_j and we have

$$\sum_{k\in N} \beta_k(p_k - c_k)\frac{\partial^2 D_k}{\partial m^2}\frac{\partial m^*}{\partial \beta_j} + (p_j - c_j)\frac{\partial D_j}{\partial m} + 2\frac{\partial D_0}{\partial m}\frac{\partial m^*}{\partial \beta_j} + (m^* - c)\frac{\partial^2 D_0}{\partial m^2}\frac{\partial m^*}{\partial \beta_j} = 0.$$

$$\frac{\partial m^*}{\partial \beta_j} = \frac{(p_j - c_j)(-\frac{\partial D_j}{\partial m})}{\sum_{k\in N} \beta_k(p_k - c_k)\frac{\partial^2 D_k}{\partial m^2} + 2\frac{\partial D_0}{\partial m} + (m^* - c)\frac{\partial^2 D_0}{\partial m^2}}. \tag{5.4}$$

By Lemma 1.2, $\frac{\partial D_j}{\partial m} < 0$ and therefore the numerator is positive.

We want the denominator of (5.4) to be negative such that $\frac{\partial m^*}{\partial \beta_j} < 0$. Using $\sigma_0 = \frac{-\partial D_0/\partial m}{D_0}$, the optimal $(p_j^* - c_j)$ from (8), and optimal $(m^* - c)$ from (12) (of the main text) in the denominator of (5.4), we have

$$\begin{aligned} & \sum_{k\in N} \beta_k(p_k - c_k)\frac{\partial^2 D_k}{\partial m^2} + 2\frac{\partial D_0}{\partial m} + (m^* - c)\frac{\partial^2 D_0}{\partial m^2} \\ = \; & \sum_{k\in N} \beta_k \cdot \frac{p_k}{\epsilon_k}\frac{\partial^2 D_k}{\partial m^2} + 2\frac{\partial D_0}{\partial m} + \frac{1 - \sum_{k\in N} \beta_k \cdot \frac{\eta_k}{\epsilon_k}}{\sigma_0}\frac{\partial^2 D_0}{\partial m^2} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{k\in N}\frac{\beta_k}{\epsilon_k}(p_k\cdot\frac{\partial^2 D_k}{\partial m^2}-\frac{\eta_k}{\sigma_0}\frac{\partial^2 D_0}{\partial m^2})+(2\frac{\partial D_0}{\partial m}-\frac{D_0}{\partial D_0/\partial m}\frac{\partial^2 D_0}{\partial m^2})\\
&= \sum_{k\in N}\frac{\beta_k\eta_k}{\epsilon_k}(-\frac{D_0}{\partial D_0/\partial p_k}\cdot\frac{\partial^2 D_k}{\partial m^2}+\frac{D_0}{\partial D_0/\partial m}\frac{\partial^2 D_0}{\partial m^2})+(2\frac{\partial D_0}{\partial m}-\frac{D_0}{\partial D_0/\partial m}\frac{\partial^2 D_0}{\partial m^2})\\
&= D_0[\sum_{k\in N}\beta_k\cdot\frac{\eta_k}{\epsilon_k}(\frac{\partial^2 D_0/\partial m^2}{\partial D_0/\partial m}-\frac{\partial^2 D_k/\partial m^2}{\partial D_k/\partial m})-(2\sigma_0+\frac{\partial^2 D_0/\partial m^2}{\partial D_0/\partial m})].
\end{aligned}
$$

Therefore the necessary and sufficient condition for $\frac{\partial m^*}{\partial \beta_j}<0$ is exactly condition (2.1). Given that $\beta_j=g_j(\alpha_j)$, $g_j'>0$, $\frac{\partial m^*}{\partial \alpha_j}$ has the same sign as $\frac{\partial m^*}{\partial \beta_j}$.■

Proof of Corollary 2.3 Condition (2.1) clearly holds when condition (2.2) does.■

Proof of Proposition 2.4 Condition (10) in the main text defines $m^*(\{p_k\}_{k\in N},\{\beta_k\}_{k\in N})$. Take partial derivative with respect to p_j and we have

$$
\begin{aligned}
&\beta_j\frac{\partial D_j}{\partial m}+\sum_{k\in N}\beta_k(p_k-c_k)(\frac{\partial^2 D_k}{\partial m^2}\frac{\partial m^*}{\partial p_j}+\frac{\partial^2 D_k}{\partial m\partial p_j})\\
&+2\frac{\partial D_0}{\partial m}\frac{\partial m^*}{\partial p_j}+\frac{\partial D_0}{\partial p_j}+(m^*-c)(\frac{\partial^2 D_0}{\partial m^2}\frac{\partial m^*}{\partial p_j}+\frac{\partial^2 D_0}{\partial m\partial p_j})=0.
\end{aligned}
$$

$$
\frac{\partial m^*}{\partial p_j}=-\frac{\beta_j\frac{\partial D_j}{\partial m}+\sum\limits_{k\in N}\beta_k(p_k-c_k)\frac{\partial^2 D_k}{\partial m\partial p_j}+\frac{\partial D_0}{\partial p_j}+(m^*-c)\frac{\partial^2 D_0}{\partial m\partial p_j}}{\sum\limits_{k\in N}\beta_k(p_k-c_k)\frac{\partial^2 D_k}{\partial m^2}+2\frac{\partial D_0}{\partial m}+(m^*-c)\frac{\partial^2 D_0}{\partial m^2}}. \tag{5.5}
$$

Note that the denominator here is exactly the same as that of (5.4) in the Proof of Proposition 2.2, which is negative whenever condition (2.1) holds.

By Lemma 1.2, $\frac{\partial D_j}{\partial m}<0$, $\frac{\partial D_0}{\partial p_j}<0$. By Slutsky symmetry of V defined in (1.8), we have $\frac{\partial^2 D_k}{\partial m\partial p_j}=\frac{\partial^2 D_0}{\partial p_k\partial p_j}$, and $\frac{\partial^2 D_0}{\partial m\partial p_j}=\frac{\partial^2 D_j}{\partial m^2}$. Therefore, when $m^*\geq c$, $\frac{\partial^2 D_j}{\partial m^2}\leq 0$, and $\frac{\partial^2 D_0}{\partial p_j\partial p_k}\leq 0$, the numerator here is also negative, which implies $\frac{\partial m^*}{\partial p_j}<0$.■

Proof of Lemma 3.1 It follows from the Slutsky symmetry of $V'(\cdot)$.■

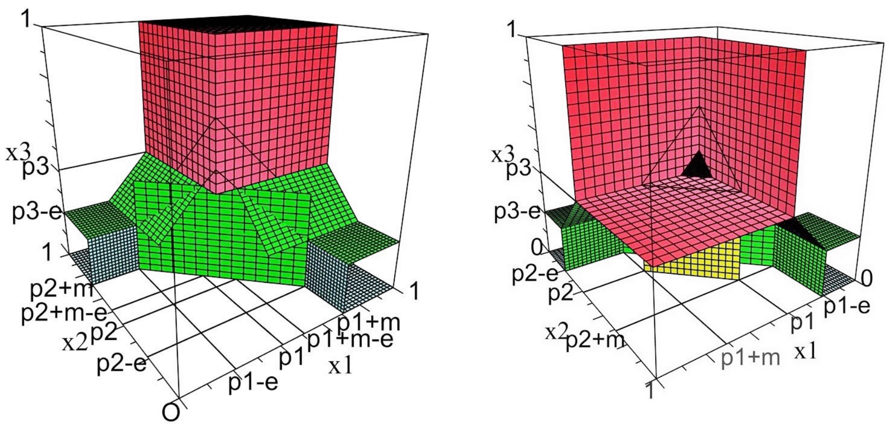
Proof of Proposition 3.2 By Definition 3 and Lemma 5.2, we first derive the allocation induced by $\mathbf{R}=(m,\mathbf{p},e,\underline{n})$, as summarized in the following lemma.

Lemma 5.4 (Allocation under Access Discount) *The allocation $\{A_J(\mathbf{R})\}_{J\subset N}$ induced by the two-part tariff with an access discount $\mathbf{R}=(m,\mathbf{p},e,\underline{n})$ in (3.1) sat-*

isfies, for any $J \subset N$*,*

$$
\begin{aligned}
A_{J\subset N,0<|J|<\underline{n}-1}(\mathbf{R}) = \quad & \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C; \sum_{j\in J} x_j \geq p_J + m; \\
& \textit{and } \sum_{k\in K} x_k \geq p_K - e, \forall K \subset N, |K| \geq \underline{n} - |J|, K \cap J = \varnothing\}, \\
A_{J\subset N,|J|=\underline{n}-1}(\mathbf{R}) = \quad & \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k \leq p_k - e, \forall k \in J^C; \sum_{j\in J} x_j \geq p_J + m\}, \\
A_{J\subset N,|J|=\underline{n}}(\mathbf{R}) = \quad & \{\mathbf{x} \in I^n | x_j \geq p_j - e, \forall j \in J; x_k \leq p_k, \forall k \in J^C; \sum_{j\in J} x_j \geq p_J + m - e; \\
& \textit{and } \sum_{h\in H} x_h - p_H \geq \sum_{k\in K} x_k \geq p_K, \forall H \subset J, K \subset N, K \cap J = \varnothing\}, \\
A_{J\subset N,|J|>\underline{n}}(\mathbf{R}) = \quad & \{\mathbf{x} \in I^n | x_j \geq p_j, \forall j \in J; x_k \leq p_k, \forall k \in J^C; \sum_{j\in J} x_j \geq p_J + m - e\}.
\end{aligned}
$$

This allocation is illustrated in the following figure.



(On the Left: View from Origin) (On the Right: View Facing Origin)

Figure 4. Allocation under Access Discount

$(n = 3, m = 0.2, p_j = 0.4, e = 0.1, \underline{n} = 2)$

A comparison between Figures 2 and 4 clearly shows the changes in allocation when the platform offers an access discount.

The partial derivative of $\pi(m, \mathbf{p}, e, \underline{n})$ with respect to e is:

$$
\frac{\partial}{\partial e}\pi'(m, \mathbf{p}, e, \underline{n}) = \sum_{j\in N} \beta_j (p_j - c_j) \cdot \frac{\partial D_j}{\partial e} + (m - c) \cdot \frac{\partial D_0}{\partial e} - D_{(|J|\geq \underline{n})} - e\frac{\partial D_{(|J|\geq \underline{n})}}{\partial e}
$$

When it is evaluated at $(m^*, \{p_j^*\}_{j\in N}, e=0, \underline{n})$, by (3.3) we have:

$$
\begin{aligned}
& \frac{\partial}{\partial e}\pi'(m^*, \{p_j^*\}_{j\in N}, e, \underline{n})|_{e=0} \\
= & \sum_{j\in N}\beta_j(p_j^* - c_j)\cdot\frac{\partial D_j}{\partial e} + (m^* - c)\cdot\frac{\partial D_0}{\partial e} - D_{(|J|\geq\underline{n})} \\
= & \sum_{j\in N}\beta_j(p_j^* - c_j)\cdot(-\frac{\partial D_{(|J|\geq\underline{n})}}{\partial p_j}) + (m^* - c)\cdot(-\frac{\partial D_{(|J|\geq\underline{n})}}{\partial m}) - D_{(|J|\geq\underline{n})} \\
= & D_{(|J|\geq\underline{n})}[\sum_{j\in N}\beta_j\frac{p_j^* - c_j}{p_j^*}(-\frac{\partial D_{(|J|\geq\underline{n})}}{\partial p_j}\frac{p_j^*}{D_{(|J|\geq\underline{n})}}) + \frac{m^* - c}{m^*}(-\frac{\partial D_{(|J|\geq\underline{n})}}{\partial m}\frac{m^*}{D_{(|J|\geq\underline{n})}}) - 1].\blacksquare
\end{aligned}
$$

Proof of Lemma 4.1 It follows from the Slutsky symmetry of V in (4.3).■

Proofs of Lemma 4.2, and Propositions 4.3 and 4.4 They are the same as those for their counterparts in the main text.■

Proof of Lemma 4.5 $\Pr[q^*(\boldsymbol{\theta}, m, \mathbf{p}) > 0] = D_0(m, \mathbf{p})$. Assumption A2 implies that given $(m, \mathbf{p})$,

$$\frac{\partial}{\partial p_j}(\frac{q_j^*(\boldsymbol{\theta}, m, \mathbf{p})}{D_0(m, \mathbf{p})}) \leq 0 \Leftrightarrow \frac{\partial q_j^*}{\partial p_j}\cdot D_0 \leq \frac{\partial D_0}{\partial p_j}\cdot q_j^* \tag{5.6}$$

By definition, $D_j(m, \mathbf{p}) \equiv \int_\Omega q_j^*(\boldsymbol{\theta}, m, \mathbf{p})dF(\boldsymbol{\theta})$ which implies $\frac{\partial D_j}{\partial p_j} = \int_\Omega \frac{\partial q_j^*}{\partial p_j}dF(\boldsymbol{\theta})$, and by taking expectations with respect to $\boldsymbol{\theta}$ on both sides of (5.6) we have

$$
\begin{aligned}
\frac{\partial D_j}{\partial p_j}\cdot D_0 &\leq \frac{\partial D_0}{\partial p_j}\cdot D_j \Longleftrightarrow \\
-\frac{\partial D_j}{\partial p_j}\cdot\frac{p_j}{D_j} &\geq -\frac{\partial D_0}{\partial p_j}\cdot\frac{p_j}{D_0} \text{ (whenever } p_j \geq 0) \Longleftrightarrow \\
\epsilon_j &\geq \eta_j.\blacksquare
\end{aligned}
$$

Proofs of Corollaries 4.6 and 4.7 They are the same as those for their counterparts in the main text.■

References

[1] Armstrong, M. (1996): "Multiproduct Nonlinear Pricing", *Econometrica*, 64(1): 51-75.