Publicization and Competition: Government Intervention in the Market for Transportation*

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Abstract

We study the optimal pricing problem in a transportation system with linear latencies, where heterogeneous commuters maximize their utility and each firm maximizes a weighted sum of social welfare and firm profit, with the weight measuring the publicization level of the firm. For a given market structure, either Monopoly or Oligopoly, we fully characterize the equilibrium of the transportation system, assuming consumers are uniformly distributed, under a serial or parallel route structure, respectively. In the serial Monopoly case, we show that the equilibrium price (flow) is decreasing (increasing) in the monopolist’s publicization level, with the socially optimal price (flow) achieved when the monopolist becomes a fully public firm. In the serial Duopoly case, we show that when a fully public firm competes against a fully private firm, the former will subsidize (by charging a negative price) and the latter will charge a price even higher than the monopolist’s price, resulting in social optimum. However, in a different serial Duopoly case where both firms are equally public and private, both firms will charge positive prices and the social optimum cannot be achieved. Our study implies that a similar publicization level in the aggregate may lead to very different equilibrium for the transportation system, if the publicization level varies across individual firms.

Keywords: Publicization, Duopoly Competition, Serial structure, Parallel structure

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1 Introduction

An industry tends to have a natural monopoly when costs display scale economies that are strong enough to make it inefficient to have more than one producer. Examples include the provision of electricity, gas, and transportation infrastructures such as railways. Although it may be efficient to have only one firm in such industries, a profit-driven monopoly without competition or regulation could exercise market power, provide lower capacity, and set higher prices. Therefore, monopolies often call for government intervention. We consider two ways that the government regulates a transportation market which is originally operated by a monopolist: taking (partial) ownership of the private firm or entering the market as a competitor. These two regulation methods are relevant in practice. One example is that each of the earliest transit routes and surface lines (stagecoach, omnibus, trolley buses, and subway) in New York was established and initially managed by a private company before 1900s. Later the government not only created the first city-run subway service in the 1930s to compete with private firms in the transportation sector, but also acquired private transit corporation and bus companies in the 1940s. Another recent example related to the government acquiring ownership is the re-municipalization of production and transport of water and energy: governments in OECD states began retrieving previously privatized and outsourced public properties and services.

We consider these two types of regulations when transportation is originally operated by a private monopoly, taking into consideration the road structures. On the one hand, in large cities such as Beijing and Singapore, subways and buses often provide parallel transportation, with one underground (subway) and the other above ground (bus). Commuters can choose either subway or bus in commuting from home to their office. One the other hand, as discussed in Kuang, Lian, Lien and Zheng (2020)[1], many transportation plans involve more than one segment. Take one daily example: urban residents may take Uber or a taxi followed by the subway to their place of employment to overcome the last mile problem caused by public transits (Hall, Palsson and Price, 2018[2]). If there is only one firm, it can always internalize road structures when making decisions. By contrast, when there is more than one firm, firms can supply transport over the entire route where they provide substitute goods (parallel structure), or each firm can serve as the sole supplier over a particular segment where they provide complimentary goods (serial structure). Road structure matters for the equilibrium result and for government interventions.

Our research questions are: given that the road structure is either parallel or serial, is it better for the government to enter the market as a competitor or to publicize the private monopoly? Given that the government decides to enter the market, thus creating a market with two suppliers, which road structure brings greater welfare gains? We analyze these questions in a model in which commuters care about both time and transport price and can decide whether or not to travel. We find that the effect of the specific regulation depends on the transportation network structure.
When the transportation network structure is serial, the government achieves social optimum more effectively by entering as a serially competing firm than exerting partial ownership. When the transport network structure is parallel, entering as a competitor is more effective than having partial ownership if the publicized firm gives more than 25% weight to profit, and vice versa if the publicized firm gives at least 75% weight to social welfare. Our analysis also shows that given that the government decides to enter the market as a direct competitor, a serial transport network structure is preferred over a parallel structure.

Our paper is related to the literature on mixed duopoly in which a public firm or a firm jointly owned by both public and private sectors competes against a private firm (Matsumura, 1998[3]; Fujiwara, 2007[4]; Ishibashi and Kaneko, 2008[5]; Li, Cai and Cai, 2019[6]) or mixed oligopoly (Merrill and Schneider, 1966[7], Harris and Wiens, 1980[8]; De Fraja and Delbono, 1989[9]; 1990[10]). Most previous papers denoted the firm jointly owned by both public and private sectors as (partially) privatized firm, but we denote such a firm as (partially) publicized firm, since we want to compare different regulations when there is a private incumbent. The key feature in previous papers and our model is the same: the objective of such a firm is a weighted average between its profit and social welfare. Similar to our analysis, many papers consider how many shares in the jointly owned firm the government should hold (Matsumura, 1998[3]; Fujiwara, 2007[4]; Matsumura and Okamura, 2015[11]). However, the majority of previous papers consider a model in which the duopolists produce substitutable commodities (Matsumura, 1998[3]; De Palma and Lindsey, 2000[12]; Brcena-Ruiz and Garzn, 2003[13]; Van Dender, 2005[14]; Matsumura and Okamura, 2015[11]). We consider the case that the routes are either substitute goods or complimentary goods.

Our paper is related to government participation in market and thus is closely related to welfare analysis on regulation. Besides involving public firms and private firms in the market mentioned above, other specific regulation instruments include tolls (Van Den Berg and Verhoef, 2011[15]; Silva, Verhoef and Van Den Berg, 2013[16]; Rouhani, Gao and Madani, 2015[17]; Song, Zhao, Jin and Sun, 2018[18]), subsidies (Feng, Zhang, Gao and Zhang, 2016[19]; Li and Cai, 2017[20]) and government guarantees (Feng, Zhang and Gao, 2015) among many other instruments. Some papers find that involving public firms or welfare-maximising pricing is better than pure private market or revenue-maximising pricing (Brueckner, 2004[22]; Verhoef and Small, 2004[23]; Matsumura and Kanda, 2005[24]; Basso, 2008[25]; Li, Cai, Feng, Xu and Cai, 2019[26]), while others find that mixed duopoly or centralization can be less efficient or leading to lower social welfare (Boardman and Vining, 1989[27]; De Palma and Lindsey, 2000[12]; Adler, Fu, Oum and Yu, 2014[28]; Zheng and Negenborn, 2014[29]). While the literature mainly focuses on welfare analysis of one particular

\[1\] Feng, Zhang and Gao (2015)[21] discuss three government guarantee types that include minimum traffic guarantee, minimum revenue guarantee, and price compensation guarantee, and show that different government guarantee types have differential effects on road quality and capacity. Quality of private toll roads refers to characteristics of the road, travel corridor, on-road or roadside service, and the impact on the environment.
instrument, we compare social welfare when the government takes ownership of the private firm or enters the market as a competitor.

Another strand of literature on government intervention focuses on ownership and discusses privatization in the past decades and recent re-municipalisation. Many governments have implemented privatization programs or outsource service to private firms in infrastructure sectors to gain efficiency and reduce financial and administrative responsibilities (Carmona 2010[30]; Tan, Yang and Guo, 2010[31]; Haralambides and Gujar, 2011[32]; Winston and Yan, 2011[33]; Rouhani, Niemeier, Knittel and Madani, 2013[34]; Chow, 2014[35]; Rouhan and Niemeier, 2014[36]; Panayides, Parola and Lam, 2015[37]; Rouhani, Gebbes, Gao and Bel, 2016[38]). Privitization has been widely studied in the literature focusing on mixed duopoly or mixed oligopoly. However, the trend has shifted recently to retrieving previously privatized and outsourced public properties and services in the collection, production and transport of water and energy in some OECD states, since the anticipated benefits of privatising public services are only achieved to a limited extent, and there is increased awareness of downsides and issues related to privatisation (Pigeon, McDonald, Hoedeman and Kishimoto, 2012[39]; Hall, Lobina, and Terhorst, 2013[40]; Busshardt, 2014[41]). Although various regulations have been studied in the literature, to our knowledge, we are the first paper to compare the government taking ownership of a private monopolist to competition between a public firm and a private firm.

We follow the literature that mainly focuses on parallel and serial networks to derive useful economic insights without having to engage in the complexity of real-world networks. A given origin-destination pair can typically be represented by parallel routes or serial links potentially operated by different providers. Parallel routes and serial links perform as substitutes and complements in transport modes, respectively. The form of the network leads to different equilibrium results. Small and Verhoef (2007)[42] show that whether the other unpriced link is parallel or serial affects the second-best pricing of other road, and they attribute this phenomenon as “network spillovers”. Acemoglu and Ozdaglar (2004)[43] show that pricing by a monopolist controlling all links in a parallel-link network always achieves efficiency with inelastic and homogeneous users, while Acemoglu and Ozdaglar (2007, IEEE)[44] show that the efficiency loss relative to the social optimum can be arbitrarily large in the presence of serial link and explains the result that when a particular provider charges a higher price, it creates a negative externality on other providers along the same path, because this higher price reduces the transportation demand along the entire path. For other related papers, please refer to De Borger and Stef Proost (2012) who provide a comprehensive literature review on serial, parallel or mixed transport network, distinguishing market structure, ownership (government, private firm or free access road), and instruments used (pricing, capacity or regulation). Recent works on serial links or complementary network include Czernyna,

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While Acemoglu and Ozdaglar (2007, IEEE)[44] do not mention specific meaning of efficiency, it is measured as the difference between users willingness to pay and delay costs in Acemoglu and Ozdaglar (2007, MOR)[45].
Van Den Berg and Verhoef (2016)[46] and on parallel links include Fu, van den Berg and Verhoef (2018)[47]. In our case, when the government enters the market as a competitor, the equilibrium results will differ under different road structures.

We also contribute to the literature on user heterogeneity. We allow commuters to have different valuations of travel, and to decide whether to take the trip or not accordingly. Incorporating user heterogeneity is important. For example, Verhoef and Small (2004)[23] show that ignoring heterogeneity in values of time causes the welfare benefits of a second-best pricing policy on one of two parallel links to be dramatically underestimated. In terms of modelling, there are in general two types of approaches. In the first general approach, the entire population of users is segmented into a number of groups or classes according to their value of time or income (Small and Yan, 2001[48]; Yang, Tang, Cheung and Meng, 2002[49]; Guo and Yang, 2009[50], 2010[51]; Fu, Van Den Berg and Verhoef, 2018[47]). The second type of approach assumes a continuously distributed value of time across users (Mayet and Hansen, 2000[52]; Verhoef and Small, 2004[23]; Light, 2009[53]; Nie and Liu, 2010[54]; Tan and Yang, 2012[55]). We follow the literature that uses a parameter to continuously measure value of time for each person, but we differ from most papers in that travellers in our model have the option not to travel. This feature of model is practical considering that employees in many companies such as Google have the option to work from home nowadays.

The remainder of the paper is organized as follows: Section 2 begins with the monopoly model where the government decides to publicize the private monopolist. Section 3 introduces a mixed duopoly model that consists of one private firm and one public firm after the government enters the market under two transport networks. We compare the results from two regulations in Section 4. We consider both publicization and competition in Section 5 using a duopoly model under two route structures. Section 6 concludes by summarizing the results, highlighting the contributions and discussing future work.

2 Publicization: The Monopoly Model

2.1 Settings

In order to focus on the role of government regulations, we analyze a simple model that is rich enough to study different types of regulations. We treat user preferences for travel as exogenously given rather than endogenously derived, and capacities as given as in Verhoef and Small (2004)[23].

We consider a model in which commuters travel from origin \( A \) to destination \( C \). The value of transport is a random variable \( \theta \) that is uniformly distributed within the interval \( [0, 1] \). People value their travel time differently, depending on their purposes for traveling or preferences even for the same travel purpose. We normalize the size of commuters as a mass of size 1. Commuters take both price and time into consideration in their utility function. For the sake of simplicity, we
assume that a commuter is willing to travel if and only if his/her value is no less than the total service fee $p$ charged by the firm plus the travel latency $f(x)$, where $x$ measures the traffic load (or traffic flow) from $A$ to $C$. Therefore, the utility for a commuter with value of transport $\theta$ and traffic load $x$ is defined as

$$u(\theta, x) = \theta - p - f(x) \quad (1)$$

We normalize the utility of the outside option (deciding not to travel, or using an alternative means of travel) as 0.

We assume that the latency is linear with respect to total traffic load $x$,

$$f(x) = t(1 + \alpha x) \quad (2)$$

where $t$ represents the idle travel time with zero traffic load. $t$ can also be interpreted as the distance of a route, or representing transportation modes of different speeds. For example, a large $t$ could represent a lengthy route, or alternatively a slower transport mode such as cycling. A small $t$ could represent either a short route, or a fast transport mode such as high speed rail. Since the commuter’s value of transport $\theta$ is in the range $[0,1]$, we focus on the reasonable situations in which $t \leq 1$. Note that if $t$ is greater than 1, then no commuters will choose to travel from $A$ to $C$. $\alpha$ is the other latency parameter, which determines how much traffic is slowed due to increased traffic flow. An interpretation of the parameter is the condition of the road, where low $\alpha$ indicates good road conditions and high $\alpha$ indicates poor road conditions.

After model settings, to gain insight into the effects of the different regulations on which this paper focuses, we follow the literature like De Borger and Van Dender (2006)[56]. We first derive price and quantity for a social welfare-maximizer, followed by a price-free case, and then provide a detailed comparison of the implications of two regulations.

### 2.2 Benchmark I: Social Optimum

When the total traffic flow is $x$, the social welfare, defined as the sum of consumer surplus and producer surplus (or firm profit), is given by

$$SW(x) = \int_{1-x}^{1} \theta d\theta - x f(x) = \frac{2x - x^2}{2} - tx - \alpha tx^2 \quad (3)$$

where we assume that commuters with higher value have priority to take the trip. Maximization of the social welfare gives us the socially optimal traffic flow

$$x_{Opt} = \frac{1 - t}{1 + 2\alpha t} \quad (4)$$

### 2.3 Benchmark II: Price-free User Equilibrium

Now we consider the price-free scenario, which is commonly studied in the transportation literature. Under such a scenario without any pricing system, the commuters can travel freely on any
route subject only to the latency of the route, thus the equilibrium traffic flow will be

\[ x_{UE} = \frac{1 - t}{1 + \alpha t} \]  

which can be easily obtained from equation (1) by letting \( p = 0 \) and \( \theta = 1 - x \).

### 2.4 Publicized Monopoly Equilibrium

We now consider the case where the government partially publicizes the monopolist. The publicized firm maximizes a weighted sum of firm profit and social welfare, with the weight \( w \) measuring profit and \( 1 - w \) measuring social welfare, indicating the publicization level. The settings for the publicization case can be illustrated by Figure 1. Note that when there is only one firm in the market, whether the structure is parallel or serial does not matter for his decision. To be more specific, the monopolist chooses a price to maximize the following objective function

\[ w \pi(x(p)) + (1 - w)SW(x(p)) \]  

Suppose that the monopolist charges a service fee of \( p \), then in equilibrium the traffic load \( x \) is determined by the marginal commuter who has value of travel \( \theta = 1 - x \) and utility \( u(\theta, x) = 0 \). Thus we have

\[ 1 - x - p - t(1 + \alpha x) = 0, \]  

which leads to

\[ x = \frac{1 - p - t}{1 + \alpha t}. \]  

Given that the firm profit is simply the product of traffic load and the price and the social welfare is defined by equation (3), solving for the firm’s optimization problem gives the equilibrium monopoly price\(^3\)

\[ p_{Mon} = \frac{(w + \alpha t)(1 - t)}{1 + w + 2\alpha t} \]  

\(^3\)For simplicity, in the benchmark models, we assume that firms’ operating costs are negligible. We can allow for non-negligible operating costs and our main results still hold.
and the equilibrium monopoly traffic flow

\[ x_{Mon} = \frac{1 - t}{1 + w + 2\alpha t} \]  

(10)

It is easy to see that the price decreases with the publicization level \( 1-w \), as \( \frac{\partial p_{Mon}}{\partial w} > 0 \), and the traffic flow increases with the publicization level \( 1-w \) as \( \frac{\partial x_{Mon}}{\partial w} < 0 \).

Consider two special cases of full publicization \( (w = 0) \) and full privatization \( (w = 1) \). When \( w = 0 \), we have \( p_{Mon} = \frac{\alpha t(1-t)}{1+2\alpha t} \) and \( x_{Mon} = \frac{1-t}{1+2\alpha t} \). When \( w = 1 \), \( p_{Mon} = \frac{1-t}{2} \) and \( x_{Mon} = \frac{1-t}{2+2\alpha t} \).

It is worth noting that when \( w = 0 \) the monopoly traffic flow coincides with the socially optimal traffic flow and the monopoly price is positive. In other words, if the social welfare maximizing government could fully regulate the transportation price, the price should be greater than zero and should be set as

\[ p_{Opt} = \frac{\alpha t(1-t)}{1+2\alpha t}. \]  

(11)

3 Competition: The Duopoly Model

In this section we consider the case where the government enters the transportation market as a directly competing firm that aims at maximizing the social welfare. Under this duopoly regime, the government and the former private monopolist independently and simultaneously operate their parts of infrastructure to attain their objectives. We consider competition under two different route structures: serial road and parallel road.

3.1 Duopoly Model under Serial Structure

Commuters would like to travel from \( A \) to \( C \) through an intermediate point \( B \). There are two firms providing transport services. Firm 1 is in charge of route \( AB \) and sets an entrance fee \( p_1 \). Firm 2, which is represented or controlled by the government, is in charge of route \( BC \) and sets an entrance fee \( p_2 \). All commuters that are willing to travel from \( A \) to \( C \) must pay \( p_1 + p_2 \) as total entrance fee. Each firm sets their price strategically but independently of the other firm, in order to maximize their objectives. For firm 1 the objective is its own profit while for firm 2 the objective is the social welfare. Such a scenario is illustrated in Figure 2.
Figure 2: Competition: Serial Structure

As mentioned earlier, the value of transport from $A$ to $C$ is a random variable $\theta$ that is uniformly distributed within the interval $[0, 1]$, and we normalize the size of commuters as a mass of size 1. A commuter is willing to travel if and only if his/her value is larger than (or equal to) the total entrance fee $p_1 + p_2$ plus the travel latency $f(x)$. The utility for a commuter is defined as

$$u(\theta, x) = \theta - p - f(x)$$

where $p = p_1 + p_2$.

As in the previous case, the latency of each segment $i = 1, 2$ is linear with respect to the total traffic load $x$,

$$f_i(x) = t_i(1 + \alpha_i x)$$

where $t_i$ indicates the trip time on a completely unoccupied route $i$. We focus on the reasonable situations in which $t_1 + t_2 \leq 1$ since no commuters will choose to travel from $A$ to $C$ if $t_1 + t_2$ is greater than 1. $\alpha_i$ determines how much traffic is slowed due to increased traffic flow on route $i$.

In a serial structure, the total latency is hence the summation of the two route segments,

$$f(x) = f_1(x) + f_2(x) = (t_1 + t_2) + (t_1\alpha_1 + t_2\alpha_2)x$$

We define $t = t_1 + t_2$ and $\alpha = \frac{t_1\alpha_1 + t_2\alpha_2}{t_1 + t_2}$, then we have

$$f(x) = t(1 + \alpha x)$$

where $t$ and $\alpha$ are sufficient for analyzing the serial structure.

3.2 Duopoly Equilibrium under Serial Structure

When the total entrance fee is $p$, assuming that the demand (or traffic flow) is $x$. The equilibrium condition requires that commuters with value greater than or equal to $1 - x$ take the trip while
commuters with value less than $1 - x$ choose outside option. Therefore, the demand is given by

$$x_{Duo}^S = \frac{1 - p - t}{1 + \alpha t}$$

(15)

which is solved by $u(1 - x, x) = 0$.

Hence the profits for firm 1 and firm 2 are

$$\Pi_1(p_1) = \frac{1 - p_1 - p_2 - t}{1 + \alpha t} p_1$$

(16)

$$\Pi_2(p_2) = \frac{1 - p_1 - p_2 - t}{1 + \alpha t} p_2$$

(17)

respectively.

The objective function of firm 1 is $\pi_1(x)$ while that for firm 2 is $SW$. Solving for the equilibrium, we have $p_1 = \frac{1 + \alpha t}{1 + 2\alpha t}$ and $p_2 = -\frac{1 - t}{1 + 2\alpha t}$. We then have

$$p_1 + p_2 = \frac{\alpha t(1 - t)}{1 + 2\alpha t} = p_{Opt}$$

(18)

Which implies that $x = x_{Opt}$.

Note that in this case we have $p_2 < 0$ and $p_1 + p_2 = p_{Opt}$, which implies that the transportation market is at the socially optimal situation. Therefore, the policy implication we can draw is that in a market of serial structure, by directly entering the market as a competing firm, the government can achieve the socially optimal traffic flow by providing a subsidy rather than charging a positive price.

### 3.3 Duopoly Model under Parallel Structure

We now analyze the competition between two firms, represented by the monopolist and the government, respectively, in a transportation route system of parallel structure. Without loss of generality, we focus on interior solutions in equilibrium.

Similarly to the setup in Section 3.1-3.2, commuters would like to travel from origin $A$ to destination $C$. There are two firms: Firm 1 takes charge of one route from $A$ to $C$ and sets an entrance fee of $p_1$; Firm 2 takes charge of an alternative route from $A$ to $C$ and sets an entrance fee of $p_2$. Once again, both firms set their prices strategically and independently to maximize their objectives. Again, for firm 1 the objective is its own profit while for firm 2 the objective is the social welfare. Such a scenario is illustrated in Figure 3.

All commuters that are willing to travel from $A$ to $C$ must choose exactly one route among the two options and pay the associated entrance fee. A commuter is willing to travel if and only if his/her value of transportation is larger than (or equal to) the entrance fee plus the travel latency. For ease of notation, we again use $t_i$ and $\alpha_i$ for latency parameters for route $i$ ($i = 1, 2$) in the parallel structure, where $t_i$ indicates the trip time on a completely unoccupied route $i$ and $\alpha_i$ is
the latency parameter representing the condition of route $t_i$.

We assume these two routes are symmetric so $t_1 = t_2 = t$ and $\alpha_1 = \alpha_2 = 2\alpha$.

Since a commuter’s value of transportation is no more than 1, we restrict our focus to the reasonable situations in which $t \leq 1$.

\[ t_1, \alpha_1 \]
\[ A \]
\[ x_1, p_1 \]
\[ t_2, \alpha_2 \]
\[ x_2, p_2 \]
\[ x = x_1 + x_2 \]

\[ u = \max \{\theta - p_1 - f(x_1), \theta - p_2 - f(x_2), 0\} \]

\[ \pi_1 = x_1 p_1 \quad \pi_2 = x_2 p_2 \]

\[ v_1 = \max \pi_1 \quad v_2 = \max SW \]

Figure 3: Competition: Parallel Structure

3.4 Duopoly Equilibrium under Parallel Structure

Suppose that in equilibrium commuters of size $x_1$ go through route 1 and commuters of size $x_2$ go through route 2. Hence, the (marginal) consumer with value $1 - x_1 - x_2$ must be indifferent between the three options: going through route 1, going through route 2, and not traveling, which implies

\[ 1 - x_1 - x_2 - p_1 - t(1 + 2\alpha x_1) = 0 \]

\[ 1 - x_1 - x_2 - p_2 - t(1 + 2\alpha x_2) = 0 \]

The above equilibrium conditions imply that the two routes from $A$ to $C$ have the same total cost

\[ p_1 + t(1 + 2\alpha x_1) = p_2 + t(1 + 2\alpha x_2) \]

\[ ^4 \text{Note that in Section 3.1-3.2, } t_i \text{ and } \alpha_i \text{ are latency parameters for segment } i \text{ in the serial structure. The parameters in the serial structure and those in the parallel structure, though represented by the same symbols, do not necessarily have the same values.} \]

\[ ^5 \text{Intuitively, one can consider the symmetric parallel structure as two sub-roads with the same road condition that were generated by dividing the original road (the serial structure) into halves. Assuming the original road has parameters } t \text{ and } \alpha, \text{ since each sub-road has the same length and half width of the original road, the parameters for each sub-road should be } t \text{ and } 2\alpha, \text{ accordingly.} \]
Then, we can express traffic flows as functions of prices:

\[ x_1 = \frac{(1 + 2\alpha t)(1 - p_1 - t) - (1 - p_2 - t)}{4(1 + \alpha t)\alpha t} \]  
\[ x_2 = \frac{(1 + 2\alpha t)(1 - p_2 - t) - (1 - p_1 - t)}{4(1 + \alpha t)\alpha t} \]  
\[ x = \frac{2 - p_1 - p_2 - 2t}{2(1 + \alpha t)} \]

where \( x = x_1 + x_2 \) denotes the total flow. Hence the profit for firm 1 is given by the following expression

\[ \Pi_1(p_1) = \frac{(1 + 2\alpha t)(1 - p_1 - t) - (1 - p_2 - t)}{4(1 + \alpha t)\alpha t} p_1 \]  

To maximize firm 1’s profit, the first order condition is

\[ \frac{\partial \Pi_1(p_1)}{\partial p_1} = p_2 - 2(1 + 2\alpha t)p_1 + 2\alpha t(1 - t) = 0 \]

Recall that the social welfare is defined as follows

\[ SW = \frac{1 - (1 - x_1 - x_2)^2}{2} - t(1 + 2\alpha x_1)x_1 - t(1 + 2\alpha x_2)x_2 \]

To maximize firm 2’s objective, which is \( SW \), we have the following first order condition

\[ \frac{\partial SW(p_2)}{\partial p_2} = \frac{2 - p_1 - p_2 - 2t}{2(1 + \alpha t)} - (1 - t) + \frac{-(2 + 4\alpha t)(1 - p_1 - t) + (2 + 4\alpha t + 4\alpha^2 t^2)(1 - p_2 - t)}{2(1 + \alpha t)\alpha t} = 0 \]

The two first order conditions jointly determine the equilibrium prices

\[ p_1 = \frac{4\alpha t(1 + \alpha t)(1 + 2\alpha t)(1 - t)}{16\alpha^3 t^3 + 28\alpha^2 t^2 + 15\alpha t + 2} \]  
\[ p_2 = \frac{2\alpha t(4\alpha^2 t^2 + 5\alpha t + 2)(1 - t)}{16\alpha^3 t^3 + 28\alpha^2 t^2 + 15\alpha t + 2} \]

Thus, we obtain

\[ p = p_1 + p_2 = \frac{2\alpha t(8\alpha^2 t^2 + 11\alpha t + 4)(1 - t)}{16\alpha^3 t^3 + 28\alpha^2 t^2 + 15\alpha t + 2} \]

\[ x_{Duo}^P = x_1 + x_2 = \frac{(8\alpha^2 t^2 + 9\alpha t + 2)(1 - t)}{16\alpha^3 t^3 + 28\alpha^2 t^2 + 15\alpha t + 2} \]

Recall in the serial duopoly case when \( w = 0 \), we have

\[ p_1 + p_2 = \frac{\alpha t(1 - t)}{1 + 2\alpha t} = p_{Opt} \]
\[ x_{Duo}^S = \frac{1 - t}{1 + 2\alpha t} = x_{Opt} \]
Compare the two values, we have
\[ x_{Duo}^P < x_{Duo}^S \] (30)

This means that in the case of parallel structure, by directly entering the market as a competing
firm, the government cannot achieve the socially optimal situation.

Next, we would like to compare the parallel duopoly with the publicized monopoly. Letting
\( x_{Mon} = x_{Duo}^P \), we obtain the following cutoff condition
\[ (8w - 2)\alpha^2 t^2 + (9w - 2)\alpha t + 2w = 0, \] (31)
which is equivalent to
\[ w = \frac{1}{4}(1 - \frac{16}{\alpha t + 2} + 8(\alpha t + 2) - 23) \] (32)

It is easy to know that for \( w > 0.25 \), we have \( x_{Mon} < x_{Duo}^P \). This means that the parallel duopoly
choice is more desirable than the publicized monopoly choice such that \( w > 0.25 \).

4 Comparison of Traffic Flows

Based on the analyses in previous sections, we have the following three propositions.

Proposition 4.1. Under the serial structure, entering the market as a competing firm generally
domines publicizing the incumbent firm, in order to achieve the socially optimal traffic flow and
price.

We have seen in Section 2.4 that the traffic flow is lower than socially optimal unless the
government fully publicizes the private firm. When the government decides to publicize the private
firm, the route structure does not matter as it can always internalize the route structure when
making decisions. Meanwhile, the equilibrium price and traffic flow shown in section 3.2 is the same
as the socially optimal case when the government enters the market under serial road structure.
Therefore, entering the market as a competing firm is more effective in achieving the social optimum
when the routes are serial.

Proposition 4.2. Under the parallel structure, entering the market as a competing firm dominates
publicizing the incumbent firm if and only if \( w > \frac{1}{4}(1 - \frac{16}{\alpha t + 2} + 8(\alpha t + 2) - 23) \), from the government’s
perspective.

We use Figure 4 to illustrate the Proposition 4.2. It shows that value of weight \( w \) increases with
the parameter \( \alpha t \), but the curve goes to the limit of 0.25. It shows that the publicized firm putting
weight of more than 25% on profit, entering the market as a competing firm is better under the
parallel structure in order to achieve the social optimum.
Proposition 4.3. Given that the government chooses to enter the market as a competing firm, the serial structure dominates the parallel structure.

Proposition 4.3 is directly derived from equation (30). This means that when one public firm and one private firm compete on the roads, the complementary transportation structure is better than the substitute structure in terms of social optimum.

5 Publicization and Competition

In previous sections, we have shown models and results when the government publicizes the private firm or enters the market. We now consider a more complicated case where the government implements both regulations.

5.1 Duopoly Model under Serial Structure

The setting of model is the similar to that in Section 3.1. The difference is the objective function. From Section 3.1, an intuitive case under both publicization and competition is that the firm operating routes from \(A\) to \(C\) is publicized, with the weight \(1 - w\) measuring the publicization level, and the other firm operating route from \(B\) to \(C\) is managed by the government. Therefore, the objective function for firm 1 is a weighted average of profit and social welfare, and that for firm 2 is social welfare. However, we further extend the case and consider a model where both firms take profit and social welfare into consideration. The setting can be illustrated by Figure 5.
5.2 Duopoly Equilibrium under Serial Structure

The objective function of firm 1 is

\[ w_1 \pi_1(x) + (1 - w_1)SW_1(x) \]  

Similarly for firm 2,

\[ w_2 \pi_2(x) + (1 - w_2)SW_2(x) \]  

When \( w_1 w_2 \neq 0 \), we have

\[ p_1 = \frac{[w_1 w_2 + (1 + \alpha t)w_1 - w_2](1 - t)}{w_1 w_2(1 - \alpha t) + (w_1 + w_2)(1 + 2\alpha t)} \]  

\[ p_2 = \frac{[w_1 w_2 + (1 + \alpha t)w_2 - w_1](1 - t)}{w_1 w_2(1 - \alpha t) + (w_1 + w_2)(1 + 2\alpha t)} \]  

and

\[ p_1 + p_2 = \frac{[2w_1 w_2 + (w_1 + w_2)(\alpha t)](1 - t)}{w_1 w_2(1 - \alpha t) + (w_1 + w_2)(1 + 2\alpha t)} \]

When \( w_1 w_2 = 0 \), we have

\[ p_1 + p_2 = \frac{\alpha(1 - t)}{1 + 2\alpha t} \]

Consider some special cases.

When \( w_1 = w_2 = 1 \), this is the duopoly with two private firms. We have \( p_1 = p_2 = \frac{1-t}{3} \), and

\[ p_1 + p_2 = \frac{2(1 - t)}{3} \]

when \( w_1 = 1, w_2 = 0 \), this is the duopoly with private firm 1 and public firm 2. We have \( p_1 = \frac{1+\alpha t}{1+2\alpha t} \) and \( p_2 = \frac{1-t}{1+2\alpha t} \), and
\[ p_1 + p_2 = \frac{\alpha t(1 - t)}{1 + 2\alpha t} \] (40)

when \( w_1 = 1, w_2 \in (0, 1) \), this is the duopoly with private firm 1 and partially publicized firm 2. We have

\[ p_2 = \frac{[w_2 + (1+\alpha t)w_2 - 1][1 - t]}{w_2(1 - \alpha t)(1 + w_2)(1 + 2\alpha t)} \] and \( p_2 = \frac{[w_2 + (1+\alpha t)w_2 - 1][1 - t]}{w_2(1 - \alpha t)(1 + w_2)(1 + 2\alpha t)} \), and

\[ p_1 + p_2 = \frac{\alpha t(1 - t) + w_2(2 + \alpha t)(1 - t)}{1 + 2\alpha t + w_2(2 + \alpha t)} \] (41)

Notice that \( \frac{\alpha t(1-t) + w_2(2+\alpha t)(1-t)}{1+2\alpha t + w_2(2+\alpha t)} \) is increasing in \( w_2 \).

When \( w_1 = w_2 = \frac{1}{2} \), this is the duopoly with two half-publicized-half-privatized firms. We have

\[ p_1 = p_2 = \frac{2(1 + 2\alpha t)(1 - t)}{5 + 7\alpha t} \] (42)

Since \( \frac{2(1+2\alpha t)}{5+7\alpha t} < \frac{2}{3} \) and \( \frac{2(1+2\alpha t)}{5+7\alpha t} > \frac{\alpha t}{1+2\alpha t} \), we know that the duopoly with two half-publicized-half-privatized firms is not as desirable as the duopoly with one private firm and one public firm.

When \( w_1 = w_2 = w \), we have

\[ p = p_1 + p_2 = \frac{2(w + \alpha t)(1 - t)}{w(1 - \alpha t) + 2(1 + 2\alpha t)} \] (43)

When \( w = 0 \),

\[ p = \frac{\alpha t(1 - t)}{1 + 2\alpha t} = p^{opt} \] (44)

To compare the price under situations where one firm is not publicized and that where both firms are publicized, we consider two cases. First, let us compare equation (41) and (43) assuming the total publicization level is the same. Let \( w_1 = 1 \) and \( w_2 = 2w - 1 \) where \( 0.5 < w < 1 \) under equation (43) and \( w_1 = w_2 = w \) under equation (43).

Equation (41) becomes

\[ p = p_1 + p_2 = \frac{(4w + 2w\alpha t - 2)(1 - t)}{4w + 2w\alpha t - 1 + \alpha t} \] (45)

and equation (43) becomes

\[ p = p_1 + p_2 = \frac{(2w + 2\alpha t)(1 - t)}{w - w\alpha t + 2 + 4\alpha t} \] (46)

Subtracting equation (46) from (45) or equation (43) from (41), the difference is increasing in \( w \) given \( 0.5 < w < 1 \). When \( w = 1 \), the two equations have the same value. When \( w = 0.5 \), equation (45) is smaller than (46). Therefore, the price is lower when one private firm is not publicized than the price when both firms are partially publicized, although the total publicization level is the same.
Second, let us consider the case where the total publicization level is not fixed or the same by comparing equation (37) and (41). Let \(w_1 = w_2 \in (0, 1)\) under equation (37), and \(w'_1 = 1\) and \(w'_2 = w_1 + w_2 - 1\) under equation (41). Then equation (37) and (38) become

\[
p = p_1 + p_2 = \frac{[(2w_1w_2 + (w_1 + w_2)\alpha t)](1-t)}{w_1w_2(1-\alpha t) + (w_1 + w_2)(1+2\alpha t)}
\]

\[
p = p_1 + p_2 = \frac{[(w'_1 + w'_2)(2 + \alpha t)](1-t)}{(w'_1 + w'_2)(2 + \alpha t) - 1 + \alpha t}
\]

Subtracting equation (47) from (48) or equation (37) from (41), the difference is negative. In other words, the price is lower when one private firm is not publicized than the price when both firms are partially publicized, even when the total publicization level is not the same.

### 5.3 Duopoly Model under Parallel Structure

Similarly to the setup in Section 3.3, commuters would like to travel from origin \(A\) to destination \(C\). There are two firms: Firm 1 takes charge of one route from \(A\) to \(C\) and sets an entrance fee of \(p_1\); Firm 2 takes charge of an alternative route from \(A\) to \(C\) and sets an entrance fee of \(p_2\). Both firms set their prices strategically and independently to maximize their objectives. Each firm has the objective of a weighted sum of profit and social welfare. Such a scenario is illustrated in Figure 6.

\[
\begin{align*}
\pi_1 &= x_1p_1 \\
\pi_2 &= x_2p_2 \\
v_1 &= \max\{x_1\pi_1 + (1-x_1)SW_1\} \\
v_2 &= \max\{x_2\pi_2 + (1-x_2)SW_2\}
\end{align*}
\]

Figure 6: Publicization and Competition: Parallel Structure

### 5.4 Duopoly Equilibrium under Parallel Structure

The objective function of firm 1 is

\[
w_1\pi_1(x) + (1 - w_1)SW_1(x)
\]
Similarly for firm 2,
\[ w_2 \pi_2(x) + (1 - w_2) SW_2(x) \] (50)

We have
\[ p_2 = \frac{2 \alpha t [-w_1 w_2 + (2 + \alpha t) w_1 + 2(1 + \alpha t) w_2 + 4 \alpha t(1 + \alpha t)](1 - t)}{-(1 + 3 \alpha t) w_1 w_2 + (4 \alpha^2 t^2 + 7 \alpha t + 2)(w_1 + w_2) + 8 \alpha t(1 + 2 \alpha t)(1 + \alpha t)} \] (51)

and
\[ p_1 = \frac{2 \alpha t [-w_1 w_2 + (2 + \alpha t) w_2 + 2(1 + \alpha t) w_1 + 4 \alpha t(1 + \alpha t)](1 - t)}{-(1 + 3 \alpha t) w_1 w_2 + (4 \alpha^2 t^2 + 7 \alpha t + 2)(w_1 + w_2) + 8 \alpha t(1 + 2 \alpha t)(1 + \alpha t)} \] (52)

As in the serial structure, we compare social welfare under situations where one firm is not publicized and that where both firms are publicized. First, we compare \( w_1 = 1, w_2 = 0 \) and \( w_1 = w_2 = 0.5 \) where the total publicization level is the same.

When \( w_1 = 1, w_2 = 0 \),
\[ p_1 + p_2 = \frac{2 \alpha t (8 \alpha^2 t^2 + 11 \alpha t + 4)(1 - t)}{16 \alpha^3 t^3 + 28 \alpha^2 t^2 + 15 \alpha t + 2} \] (53)

When \( w_1 = w_2 = 0.5 \),
\[ p_1 = p_2 = \frac{2 \alpha t [-\frac{1}{4} + \frac{1}{2} (2 + \alpha t) + (1 + \alpha t) + 4 \alpha t(1 + \alpha t)](1 - t)}{\frac{1}{4}(1 + 3 \alpha t) + 4 \alpha^2 t^2 + 7 \alpha t + 2 + 8 \alpha t(1 + 2 \alpha t)(1 + \alpha t)} \] (54)

and
\[ p_1 + p_2 = \frac{2 \alpha t (8 \alpha^2 t^2 + 11 \alpha t + \frac{7}{2})(1 - t)}{16 \alpha^3 t^3 + 28 \alpha^2 t^2 + \frac{57}{4} \alpha t + \frac{7}{4}} \] (55)

Comparing equation (55) and (57), the former is larger than the later. Since equation (21) shows that traffic flow is a decreasing function in \( p_1 + p_2 \), it implies that regulation with \( w_1 = w_2 = 0.5 \) leads to higher social welfare than that with \( w_1 = 1, w_2 = 0 \). This is different from the results under serial structure.

Second, let us consider the case where the total publicization level is not the same.

Under \( w_1 = w_2 \in (0, 1) \),
\[ p_1 + p_2 = \frac{2 \alpha t(1 - t)[-2w_1 w_2 + (2 + \alpha t)(w_1 + w_2) + 2(1 + \alpha t)(w_1 + w_2) + 8 \alpha t(1 + \alpha t)]}{-(1 + 3 \alpha t) w_1 w_2 + (4 \alpha^2 t^2 + 7 \alpha t + 2)(w_1 + w_2) + 8 \alpha t(1 + 2 \alpha t)(1 + \alpha t)} \] (56)

When \( w'_1 = 1 \) and \( w'_2 = w_1 + w_2 - 1 \),
\[ p_1 + p_2 = \frac{2 \alpha t(1 - t)[-2w'_1 w'_2 + (4 + 3 \alpha t)(w_1 + w_2) + 8 \alpha t(1 + \alpha t)]}{-(1 + 3 \alpha t) w'_1 w'_2 + (4 \alpha^2 t^2 + 7 \alpha t + 2)(w_1 + w_2) + 8 \alpha t(1 + 2 \alpha t)(1 + \alpha t)} \] (57)

Comparing two equations, equation (57) is larger than equation (56). This implies that traffic flow or total welfare is larger when both firms are publicized, considering that the publicization level is the same. Depending on whether the publicization level is the same, we have different results from parallel structure and serial structure.
6 Conclusion

In transportation arrangements, a private monopolist often has initial ownership over the transport route, while the government may seek to enhance social welfare using a regulatory policy. Two natural interventions arise, which are acquiring partial ownership of the transport route by the government, and direct competition with the private firm through market entry. We analyze the relative effectiveness of each of these policies, which may depend on the transportation system structure.

Our analysis shows that indeed the government’s objective is most ideally served by different regulatory strategies depending on the structure of the transport system. Under the serial structure, entering the market as a competing firm dominates publicizing the incumbent firm. Under the parallel structure however, entering the market as a competing firm dominates publicization. Furthermore, given that the government chooses to enter the market as a competing firm, the serial structure dominates the parallel structure.

There are several potential directions for future work. Extensions of this framework could include a dynamic setting in which firms and government interact over time, and commuters may behave adaptively or with limited foresight to transport system conditions. The model can also be extended to apply practically to more complex transport network structures, in order to understand the government’s best strategies for social welfare maximization under competition versus regulation with one or more private firms.
References


