Outsiders, Insiders and Interventions in the Housing Market[*]

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Abstract

In the wake of China's "great migration," many cities, including Beijing and Shanghai, restrict some residents from owning housing, forcing them to rent. We build a model studying motivations for and effects of ownership-restricting policies. When some agents are prohibited from purchasing housing, competitive equilibrium further punishes renters, failing to attain a "second-best" that maximizes welfare subject to the policy's intended constraint. We then consider real estate taxation, a hotly debated topic in China, currently undergoing reform. We show that positive taxes on housing transactions can help mitigate the inefficiency caused by restricted ownership, but only by introducing a new distortion. Meanwhile, subsidizing rental transactions could, in theory, restore the second-best, but only by diverting public funds away from other uses.

Keywords: Ownership Restriction, Sale versus Rental, Taxation, Chinese Housing, *Hukou*, Second-Best

JEL Codes: D41, D45, H21, H30

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1 Introduction

China's "great migration" has led the Middle Kingdom's urban population to grow by half a billion people over the past few decades. Indeed, the number of domestic migrants in China exceeds the estimated 232 million international migrants throughout the rest of the world (Gardner, 2014). This mass rural-to-urban migration is often associated with China's "economic miracle" that has taken place over the same period, lifting hundreds of millions out of extreme poverty. Although the flow of migrants to the largest cities has subsided, it has, nevertheless, given rise to significant divisions between longstanding residents and recent arrivals (Gregory and Meng, 2018).

One such division lies in the housing markets of at least 50 of China's largest cities, including Beijing and Shanghai. Ever since 2010, the governments of these cities have enacted policies restricting who can purchase real estate. More specifically, the Chinese *hukou*,[1] or household registration system assigns each citizen to a specific geographic area. As the aforementioned migration numbers suggest, enforcement that people physically remain in their assigned areas is extremely lax. However, under these policies, the *hukou* system significantly affects people's ability to buy real estate. So, for instance, holders of Beijing *hukou*, often those who were born in that city, are entitled to purchase one unit of housing (of any particular size/value) to live in, as well as an extra unit that may be used to rent out. On the other hand, people living in Beijing without a local *hukou* cannot purchase any real estate until they can prove that they have paid local taxes for multiple years.

When they are spoken about in the press, the motivation typically given for such policies is to "curb prices" in a hot market.[2] However, aside from their possible effects on the overall trajectory of housing prices, a number of separate, more microeconomic questions arise regarding the motivation for and the effects of housing ownership restrictions. What kinds of allocative inefficiencies do such restrictions lead to? Whom do they benefit and whom do they harm? Moreover, Chinese officials have actively discussed reforming the ways in which real estate is taxed (see below). Does conventional wisdom regarding different forms of real estate taxation apply in an environment in which ownership restrictions are in place? Finally, in seeking to make the housing market function more smoothly, which kinds of frictions should one target?

The simple, static,[3] general equilibrium model we develop to shed light on these issues raises three main points: first, a basic economic mechanism resulting from restricted ownership drives an *inefficiently small share of the housing stock to be rented out*. Second, due to a key difference between frictions incurred by landlords and tenants, policies designed to lesson the former may be relatively more promising in this context. Third, in such an environment, transaction

[1] Pronunciation: "who-co"

[2] See, among many possible examples, Soares (2014) and Xinhua (2017a).

[3] While our baseline model is static, an extension in Subsection 5.2 considers dynamics in a two-period setting.

taxes discouraging sale or favoring rental can be corrective, but each with its own costs.

In the model, if offered the choice between purchasing and renting housing at equivalent prices, agents would (at least weakly) prefer to purchase. Meanwhile, owners of units who do not wish to occupy them prefer to sell for given price, rather than manage them as rental properties while receiving an equal stream of rental revenue. Thus, the rental market, which would otherwise be inactive, springs up as a result of the government policy restricting some agents from owning housing.

The most straightforward effect of the ownership restriction policy is that it harms non-*hukou* holders, who are forced to rent rather than own the homes they occupy. Moreover, the policy has a split impact on those who are authorized to own real estate. On the one hand, it helps those who, under the *status quo*, do not own housing by allowing them to acquire it more cheaply. On the other hand, for those who already own housing, it diminishes the demand for their surplus units, thus lowering the value of their endowments.

More subtly, when the government restricts some agents from owning real estate, competitive equilibrium is no longer efficient, even in the "Second-Best" sense of Lipsey and Lancaster (1956). In other words, keeping in place the constraint that non-*hukou* holders must rent the housing in which they live, the prices that arise at equilibrium do not maximize total surplus. This inefficiency stems from the fact that, for owners of excess housing (beyond what they wish to occupy) to be willing to put their property on the rental market, rather than to sell it, they must be compensated for the management frictions they will incur. However, such management frictions vary with the market value of the rental property. As a result, property owners who put their real estate on the rental market do not internalize the entire impact of this action. This leads to overconsumption of housing by those who are authorized to own it, and underconsumption by those who must rent.

Together, these findings call to attention to an interesting tradeoff underlying ownership restriction policies that seems orthogonal to the aforementioned, widely cited objective of maintaining price stability in the housing market. In particular, the market's failure to attain the Second-Best suggests that a city government that chooses to implement such restrictions does so, in part, to favor the interests of the relatively less-well-off natives over both wealthier natives and (often poorer) recent arrivals, at the expense of allocative efficiency.

The allocative inefficiency that arises when ownership restrictions are in place raises a host of questions about the merits of different forms of taxation in such an environment. An interesting feature of the Chinese housing market is that, in most locations, there is no property tax, although there have been rumblings that this may change.[4] There are, however, transaction tax levied when property changes hands from one party to another and when it is rented out.

[4]In 2011, experimental property tax regimes were adopted in Shanghai and Chongqing (Cao and Hu, 2016). On March 5, 2018, Premier Li Keqiang issued a report advocating for the wider adoption of a property tax system (Gopalan, 2018).

Using our model, we analyze the impact of both property taxes and taxes on sales and rental transactions.

In line with a standard result discussed by Feldstein (1977), when the stock of housing is fixed, property taxation has no impact on the equilibrium allocation of housing or on total surplus. Meanwhile, when the stock of housing is endogenous, property taxes lower the amount supplied at equilibrium, but they do not differentially affect the amount consumed by the various types of agents. Transaction taxes, on the other hand, give rise to much more novel analysis.

In particular, we show that imposing a small sales tax is always welfare-improving. This is because such a tax encourages rental. Thus, it reduces the gap, which, at equilibrium, is inefficiently large, between the aggregate amount of housing consumed by *hukou*-holding ("insider") owners and non-*hukou*-holding ("outsider") renters. However, the introduction of a sales tax also brings about a separate distortion within the group of insiders. Specifically, it discourages the insiders who initially hold a large endowment of housing from selling a sufficiently large fraction of these holdings to other, less well-endowed insiders. Consequently, the optimal sales tax is positive; it solves the tradeoff brought about by these two distortions but fails to restore the Second-Best outcome.

Unlike a sales tax, a rental tax does not introduce such a wedge between the consumption levels of the endowed and unendowed insiders. Consequently, it can restore the Second-Best. However, in order to do so, it must be negative; i.e., it must be a subsidy. This is because, in its absence, the equilibrium amount of rental is inefficiently low. Thus, although such an instrument is appealing from a theoretical perspective. In our context, however, it could be practically unappealing, as it involves using public funds to directly favor well-endowed *hukou* holders and non-*hukou* holders, at the expense of less-wealthy *hukou* holders. As such, it would seem to visibly work against the interests of the group that the ownership restriction policy, itself, is intended to favor. In addition, rental subsidies would create incentives to game the system for the sole purpose of receiving the subsidy. Currently, Beijing and other Chinese cities imposes positive transaction taxes on both sales and rental transactions.

In sum, our model

- suggests city governments' possible finer-grained motivations for adopting ownership restriction policies, beyond merely curbing housing prices,
- reveals a further distortion arising in such settings that excessively discourages rental,
- shows that management frictions incurred by landlords drive this distortion, not rental frictions incurred by tenants,
- indicates that transaction taxes can mitigate this inefficiency but only by either introducing a new allocative distortion or spending public funds.

These results can help to inform, in particular, two areas of housing policy in China. First, they are relevant for ongoing discussions about how to improve the real estate rental market. An especially relevant example of such efforts is the December 2017 conference, held by the Chinese State Council, which advocated for mechanisms to promote rental efficiency. One proposal that emerged was for the establishment of better platforms to intermediate landlord-tenant matching, to better deal with conflicts between landlords and tenants, and to improve tenants' rights (Xinhua, 2017b).

Our model shows that, so long as restricted ownership is in place, it is particularly important to focus on alleviating frictions that are proportional to the total *value*, rather than volume, of rental inventory on the market. Whereas both landlords and tenants likely incur costs that are proportional to volume (e.g, matching, transferring/receiving rent, etc.), landlords are more likely to also incur frictions proportional to property value, as the potential for loss due to damage scales in this dimension. Thus, even if tenants are the intended beneficiaries of the aforementioned efforts to improve rental, it may be worthwhile to focus attention on measures that, at a superficial level, appear to be more helpful to landlords, in order to increase supply of rental units.

Second, standard thinking about the merits of different forms of real estate taxation should be adapted to take into account the effects of restricted ownership. Following the pilot property tax programs, mentioned above, in Shanghai and Chongqing, there have been growing calls for broader adoption of ownership-based taxation. In addition to the comments of Premier Li, cited in footnote 4, officials from the tax authority have claimed that real estate taxation is more effective when it is ownership-based rather than the current, transaction-based practice (Xiao, 2017). Others have said that the two forms complement each other (Reuters, 2010).

Our results appear incompatible with the former of these two views. Under the restricted ownership policy, even without any taxes, the economy fails to obtain the Second-Best. Thus, the conventional argument for avoiding transaction-based taxes, because they more directly distort incentives,[5] does not necessarily apply. Instead, we find that properly chosen transaction taxes improve incentives. The latter view, advocating complementarity between property and transaction taxes is more compatible with our findings. In particular, we find that rental subsidies have the potential to restore the Second-Best, but they draw on public funds. These could, in theory, be raised via a property tax in a non-distortionary way. However, as we discuss in Subsection 5.3, where we extend the model from its simple, quasi-linear baseline to a version that allows for wealth effects, there are two caveats. First, we believe a good degree of caution is warranted before adopting such fiscal policy as a corrective to the effect of the initial, ownership restriction policy. Second, the relative merits of transaction-based taxes and subsidies depend on one's view of the more general macroeconomic effects of government

[5]See, e.g., Mirrlees and Adam (2010, 2011).

spending, itself a topic of debate.

Related literature. The stylized model that we develop in this paper to study certain facets of the Chinese housing market contrasts with most literature on the topic; most work in this area covers more "macro" aspects (e.g., Fang et al. (2016), Glaeser et al. (2017), Han et al. (2018)), especially the evolution of housing price levels over time, relative to other assets. Nevertheless, a recent stream of empirical papers focuses particularly on the effect of ownership restriction policies. This includes Wu et al. (2012), which points out China's high sale-to-rental price ratios in the period before the adoption of ownership restriction policies and Sun et al. (2017), showing a drop in sales prices and sales-to-price ratios once the policies took effect. Cao et al. (2015) and Du and Zhang (2015) also find the policies to be effective in curbing housing prices; Cao et al. (2018) find that such policies discourage demand for investment in housing.[6] Li et al. (2017) and Chen et al. (2018) both use simulation techniques to predict the future dynamics of housing prices in the Chinese setting with restricted ownership.[7]

Literature on real estate taxation in China includes Bai et al. (2014), Cao and Hu (2016), and Du and Zhang (2015). These articles do not, however, observe variation in transaction tax policies and thus do not have direct bearing on the primary focus of Section 4 in our paper. The aspects of the housing market that we study are motivated, broadly speaking, by China's urban-versus-rural divisions. Meng and Zhang (2001) is a pioneering work on this topic, Song et al. (2012) models Chinese urbanization, and Gregory and Meng (2018) provides a recent update on the topic.[8]

Using a stylized model to study specific public policy questions in an urban context follows in the spirit of works such as Chen and West (2000) and Ushchev et al. (2015). Given our aim of cleanly shedding light on a set of potentially complicated economic mechanisms, we intentionally ignore many of the myriad factors that also influence housing markets, especially in the baseline model described in Section 2. A vast literature addresses different topics that could potentially interact with the issues we address in interesting ways. This includes, especially, work comparing ownership and rental of real estate (see, e.g., Smith (1974); Rosen and Smith (1983); Weiss (1978); Henderson and Ioannides (1983); Flavin and Yamashita (2002); Himmelberg et al. (2005).[9] The aim of this literature differs from ours in that it seeks to explain different individuals choices to own or rent housing, whereas we take as given a general preference to own, which has been documented in China (Wei and Zhang, 2011; Hu, 2013) and study the effects of forced rental. Finally, our work builds on classic studies of property and commodity taxation (e.g., Feldstein (1977), Atkinson and Stiglitz (1976), etc.)

[6]In Subsection 5.2, we point out ways in which our model's predictions appear to match these papers' findings.

[7]In the U.S. context, Diamond et al. (2019) empirically study the effects of rent controls (i.e., rental price caps).

[8]Lui and Suen (2011) study, in the context of Hong Kong, the effects of housing interventions on internal mobility.

[9]Also somewhat related is Bulow (1982) and subsequent literature, which, in examining the Coase conjecture, compares sale and rental of a durable good when, in contrast to our setting, a supplier has market power.

2 The Model

Consider the following baseline model of a city's housing market, capturing certain key aspects of cities such as Beijing and Shanghai. A static exchange economy features N agents, each of whom falls into one of two categories: "outsiders," referred to using subscript o, and "insiders," referred to with subscript i. For our purposes, the difference between outsiders and insiders is that the law prohibits the former from owning housing, whereas such rules do not apply to the latter group. One may think of outsiders as migrants to the city, who lack the residency permit necessary to purchase a home or apartment.

Members of the latter group, on the other hand, hold such a permit, either by virtue of having been born in the city or by having obtained one upon migrating (for instance, thanks to suitable employment). Among the insiders, some are said to be "endowed," meaning that they arrive at the market already owning housing. The complementary set of insiders are, by contrast, "unendowed." Fomally, $N = n_o + n_i$, and $n_i = n_u + n_e$, where subscripts u and e refer to unendowed and endowed insiders, respectively. All endowed insiders begin with an equal share, h_e, of the total housing, $H = n_e h_e$.

On the demand side, agents have quasi-linear utility, and the benefit they perceive from occupying a given amount of housing varies, depending on whether they own or rent (we assume they cannot do both). An agent who lives in owned property receives utility $u(x) + z$, where $x \geq 0$ denotes a quantity of owned housing, and z denotes the quantity consumed of the numéraire good, "money." An agent who lives in rented property receives utility $\theta u(y) + z$, where $y \geq 0$ denotes a quantity of rented housing.

In view of the fact that, in practice, ownership restrictions appear to pose a significant binding constraint on outsiders' behavior, we assume that occupying a given rented home offers an experience that is no better and, perhaps, worse than owning and occupying the same home.[10] For instance, renters have fewer options than owners do about how a property may be used or renovated. Also, ownership can provide a sense of security and social caché. Thus, $\theta \in (0, 1]$, and this parameter measures the degree to which renting property successfully approximates the experience of owning. The function $u(\cdot)$ is strictly increasing, strictly concave, and three times continuously differentiable. We sometimes consider preferences with functional forms, such as $\log(\cdot)$, that may give rise to negative output when their arguments are sufficiently close to zero, but, for simplicity, we restrict attention to regions in the domain where the output of $u(\cdot)$ is positive, thus implying that $u(x) \geq \theta u(x)$.

Regarding supply, p denotes the "sale price" of one unit of housing, and r denotes the "rental price." Given the static nature of the model, r should be interpreted as the present discount value of the entire stream of gross rental income that a landlord would receive from

[10] Also, see Wei and Zhang (2011) and Hu (2013), which provide empirical evidence supporting the view that people in China prefer to live in homes that they own, holding fixed other factors.

renting out one unit of housing. A property owner who decides to sell x units thus receives px in money, whereas renting out y units brings in λry. Parameter $\lambda \in (0, 1)$ is analogous to θ on the supply side, and thus $(1 - \lambda)$ measures the monetary and psychic costs or "frictions" of managing property.

Note that our specification considers management frictions to be proportional to the value of the property. In reality, some frictions have this feature (e.g., risk of property damage taken on by the landlord), while others are more likely to be constant per-unit rent out (e.g., the hassle of depositing each month's rent into a bank account). The former type of frictions play an important role in our model, whereas the latter do not. Thus, in the main text, we set per-unit management frictions to be zero and then include them in the general model of Appendix B. We analyze Walrasian Equilibrium in the model. To facilitate this, we assume all agents have sufficient wealth and $u(\cdot)$ is concave enough so that budget constraints can be ignored.

Beyond the baseline model described above, we provide a range of extensions and generalizations, which are the focus of Section 5 and the appendices. In particular, that section considers (and shows robustness of our results under)

- endogenous migration of outsiders to the city (5.1);
- a dynamic specification, taking into account the potential for outsiders to become insiders following a waiting period (5.2);
- general specifications of utility that allow for wealth effects (5.3);
- richer heterogeneity among agents, both in their preferences for ownership versus rental and in their initial endowments of housing (5.4).

3 The Impact of Ownership Restriction

This section analyzes the impact of a policy that prohibits outsiders from owning housing. To do this, it first considers the benchmark in which all agents are free to rent or own real estate. It then compares the equilibrium when the ownership restriction on outsiders is in place to two different benchmarks that also satisfy such a restriction. In doing so, it first identifies the crucial condition that determines whether maximizing total utility from consumption and reducing management frictions pose a tradeoff or are aligned with one another. It then shows that equilibrium under-allocates housing to outsiders, who rent, while also failing to minimize management frictions.

3.1 No Restrictions: First-Best

When all agents can own housing, it is straightforward to see that, because of the frictions associated with rental, that market will be inactive, and any transaction that take place are sales. The equations $y^A = 0$, $x^A = H/N$, and $p^A = u'\left(x^A\right)$ characterize the equilibrium,[11] under which "utilitarian" total welfare reaches its maximal level. Note that all agents consume the same amount of housing, and, therefore, endowed insiders are net suppliers, whereas outsiders and unendowed insiders are net demanders.

3.2 Restricted Ownership

We now consider two benchmarks that satisfy the restriction against outsiders owning housing and that help illustrate the relevant forces in the model. The first benchmark arises when a hypothetical authority maximizes total surplus in the economy. This outcome, labeled B and henceforth referred to as the "Second-Best," solves

$$\max_{x,y} \quad \underbrace{n_i u(x) + n_o \theta u(y)}_{\text{utility from housing}} \quad - \quad \underbrace{n_o (1-\lambda) r y}_{\text{management frictions}}$$

$$\text{subject to} \quad n_i x + n_o y = H, \;\; \theta u'(y) = r, \;\; u'(x) = p.$$

Plugging in the constraints, this can be rewritten as

$$\max_{y} \quad n_i u\left(\frac{H - n_o y}{n_i}\right) + n_o \theta \left(u(y) - (1-\lambda) u'(y) y\right),$$

giving solution

$$u'\left(x^B\right) = \theta u'\left(y^B\right) - (1-\lambda)\,\theta\left(u'\left(y^B\right) + y^B u''\left(y^B\right)\right). \tag{1}$$

The second benchmark, labeled C and referred to as "Consumption-Optimal," maximizes consumer surplus derived directly from the consumption of housing, ignoring management frictions. It solves $\max_{x,y} \; n_i u(x) + n_o \theta u(y)$, subject to $n_i x + n_o y = H$, giving rise to

$$u'\left(x^C\right) = \theta u'\left(y^C\right). \tag{2}$$

Note that equation (2) implies that $u'\left(x^C\right) \leq u'\left(y^C\right) \Leftrightarrow x^C \geq y^C$, meaning that each insider consumes at least as much housing as each outsider.

Comparison. Lemma 1 compares these two benchmarks using the following definition.

[11] Technically, at equilibrium, r takes on some value high enough to give rise to zero rental demand and low enough to induce zero rental supply. $r = p$ satisfies this requirement in an intuitive way, because for a given x, $\lambda p x < p x$ and $\theta u(x) - p x \leq u(x) - p x$.

Definition 1. *Let* $r(y) \equiv \theta u'(y)$ *denote* Inverse Demand *for Rental.*

Definition 2. *Let* $MR(y) \equiv r(y) + y\frac{dr}{dy}$ *denote the rental market's* Marginal Revenue*. That is, it measures the additional pre-management-friction revenue spent per outsider on rental housing when the per capita stock of housing devoted to rental,* y*, increases marginally.*

Lemma 1. *When* $MR\left(y^B\right) > 0$*, the authority favors sales over rental, under the Second-Best regime, compared to the outcome under the Consumption-Optimal regime. When* $MR\left(y^B\right) < 0$*, the opposite is true, and, when* $MR\left(y^B\right) = 0$*, the allocations under the two regimes are identical. Formally,*

$$sign\left\{\theta u'\left(y^B\right) - u'\left(x^B\right) = r^B - p^B\right\} \; = \; sign\left\{x^B - x^C\right\} \; = \; sign\left\{y^C - y^B\right\} \; = \; sign\left\{MR\left(y^B\right)\right\}.$$

Starting from $u'\left(x^C\right) = \theta u'\left(y^C\right)$, why would a switch to the Second-Best regime lead to an increase in the share of housing devoted to sales if and only if marginal rental revenue from rental is positive? This is because aggregate management frictions vary with the total value of the property that is put up for rental. In what is, perhaps, the more intuitive case, where marginal rental revenue is positive, the regime that *does* take into account management frictions seeks to avoid these by inducing less rental than the regime that does not take them into account. However, when marginal rental revenue is negative, the way to lower aggregate management frictions is, in effect, to flood the market with rental property, driving down its price and thus the magnitude of value that dissipates as a result of them.

Equilibrium Under Restricted Ownership. At equilibrium, outsiders' only option is to rent housing, and they choose a quantity such that $\theta u'(y^*) = r^*$. In principle, insiders could choose either to rent or to own, but, so long as the sales price is less than the rental price, they prefer to own. In this case, they choose a quantity satisfying $u'(x^*) = p^*$. Moreover, this preference for ownership on the part of insiders can be verified thanks to the indifference condition for endowed insiders between selling and renting out, $p^* = \lambda r^*$. This must hold in order to simultaneously elicit a positive supply of housing in both the rental and sales markets. Combining these, at equilibrium, we have that

$$u'(x^*) = p^* = \lambda\theta u'(y^*). \tag{3}$$

Note that equation (3) implies that, at equilibrium, the marginal utility to an outsider of occupying more housing, by renting more, is greater than the marginal utility to an insider of occupying more, by owning more. That is, $\theta u'(y^*) > u'(x^*)$, which also implies that $y^* < x^*$.

How does equilibrium compare to the Second-Best and Consumption-Optimal benchmarks? Propositions 1 and 2 make these comparisons.

Proposition 1. *Compared to benchmark C, which maximizes total utility from housing consumption, equilibrium allocates less housing to rental and more to ownership. Moreover, these outcomes satisfy*

$$y^* < y^C \leq x^C < x^*. \tag{4}$$

Proposition 2. *Compared to benchmark B, which maximizes total surplus, equilibrium allocates less housing to rental and more to ownership. That is, $y^* < y^B$ and $x^B < x^*$, but a complete ranking cannot be guaranteed.*

These comparisons between equilibrium under restricted ownership and the two benchmarks highlight the following two points. First, and most obviously, under this form of restriction on housing ownership, competitive equilibrium is inefficient. Second, and more interestingly, the inefficiencies that arise stem from both the demand and the supply sides of the market in a rather intricate way. By ignoring management frictions, the Consumption-Optimal benchmark illustrates cleanly the way that, when the only welfare concern is to maximize utility from housing consumption, equilibrium gives rise to an allocation under which insiders are inefficiently "subsidized."

Meanwhile, the Second-Best benchmark fully integrates the objective of minimizing management frictions. Its comparison to equilibrium reveals that, even when the benchmark takes management frictions into account, equilibrium *still* inefficiently subsidizes ownership. Seen from this perspective, we can now interpret Lemma 1 as offering the criterion that determines the direction in which management frictions push. On the one hand, when marginal rental revenue is positive, equilibrium's inefficient subsidy of ownership serves, at least, to reduce management frictions. On the other hand, when marginal rental revenue is negative, this ownership subsidy also exacerbates management frictions. Indeed, as the last sentence in Proposition 2 indicates, if $MR\left(y^B\right)$ is sufficiently negative and θ sufficiently close to one, the Second-Best may give rise to $x^B < y^B$, that is, a higher level of housing consumption for renters than for owners. Figure 1 illustrates this point.

Who Benefits from Ownership Restrictions? We now briefly take stock of the basic impacts of (and thus possible motivations for) a policy of ownership restriction. Compared to the free market configuration, discussed in Subsection 3.1, it is straightforward to see that ownership restrictions have a negative impact on outsiders. Do these restrictions make insiders better off? The key point is that this varies across the two types of insiders. Endowed insiders are net suppliers of housing. When the authority imposes ownership restrictions on outsiders, this imposes frictions (via θ and λ) that effectively make housing a less valuable good. This reduces the demand for housing from outsiders, thus pushing down total demand for housing, making endowed insiders worse off. On the other hand, ownership restrictions imposed on outsiders have a positive impact on unendowed insiders. This is because, in an unrestricted market, each

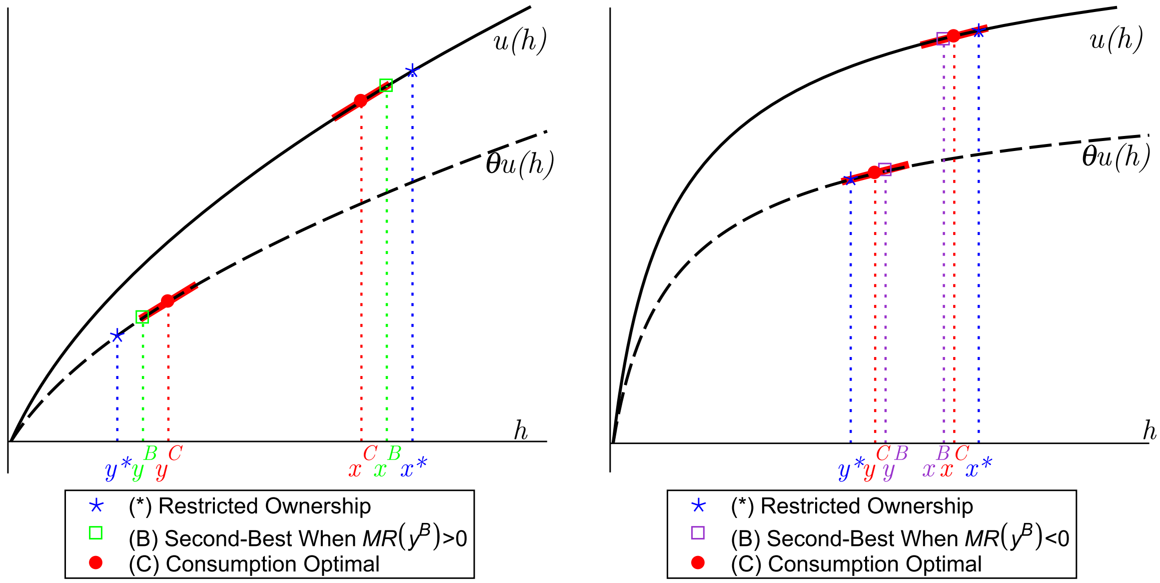

Figure 1: *Marginal utility levels under the Second-Best and Consumption-Optimal regimes and at equilibrium. The left panel illustrates the case where* $MR\left(y^B\right) > 0$, *and the right panel illustrates the case where* $MR\left(y^B\right) < 0$.

outsider's demand for housing is as large as each insider's, whereas, when the restrictions are in place, a given outsider demands less than a given insider. From an unendowed insider's perspective, this results in less competition for the fixed stock of housing.

A Useful Class of Utility Functions. Before moving on to analyze the impacts of taxation, note that a particularly apt class of utility function for our analysis are those of the Isoelastic form,

$$u(x) = \begin{cases} \frac{x^{1-\eta}-1}{1-\eta}, & \eta \neq 1 \\ \log(x), & \eta = 1 \end{cases}, \tag{5}$$

where $\eta \in (0, \infty)$. For a general function, $u(\cdot)$, the elasticities of sales and rental demand are given by

$$\frac{p}{-x\frac{dp}{dx}} = \frac{u'(x)}{-xu''(x)} \quad \text{and} \quad \frac{r}{-y\frac{dr}{dy}} = \frac{\theta u'(y)}{-y\theta u''(y)},$$

respectively. This means that, for a given y, $MR(y) > 0$ if and only if, locally, the elasticity of demand for rental is greater than 1. Under this specification (said, in other contexts, to exhibit Constant Relative Risk Aversion), elasticity is constant, and its inverse is measured by η. That is,

$$\eta = \frac{-yu''(y)}{u'(y)}.$$

Consequently, it is straightforward to see that, under Isoelastic demand, when $\eta \in (0,1)$, i.e., demand is relatively elastic, increasing the share of housing devoted to rental, and thus lowering r, always increases management frictions. In constrast, when $\eta > 1$, i.e., demand is relatively inelastic, such a change always decreases management frictions. Thus, outcomes under the Second-Best and Consumption-Optimal benchmarks obey the following relationships:

$$\begin{cases} x^B > x^C & \text{and} \quad y^B < y^C \quad \text{if } \eta \in (0,1) \\ x^B = x^C & \text{and} \quad y^B = y^C \quad \text{if } \eta = 1 \\ x^B < x^C & \text{and} \quad y^B > y^C \quad \text{if } \eta > 1 \end{cases}.$$

In sum, Isoelastic demand has the desirable property that adjusting its parameter, η, transparently impacts a crucial property of the model, namely, the sign and strength of the objective to reduce management frictions, relative to the objective to maximize consumption utility. In addition, it gives rise to closed-form solutions to the model in a number of different taxation regimes. We, therefore, make use of this functional form in the next section.

4 The Effects of Taxation Under Ownership Restriction

4.1 Property Tax

First consider a property tax (i.e., taxation on ownership of housing) when outsiders face ownership restrictions. Under such a regime, after all transactions have occurred, the authority levies a tax on any agent who owns housing that is proportional to its monetary value. In this environment, the following is true.

Claim 1. *A property tax has no impact on equilibrium housing consumption; it merely extracts wealth held initially by endowed insiders.*

This result follows the classical Ricardian argument, which Feldstein (1977) summarizes (and relaxes), that "a tax can be shifted only by reducing the supply of the taxed activity." We now concentrate of transaction-based taxes, which do have such an effect and whose impacts are thus more interesting in this quasi-linear setting.

4.2 Sales Tax

Now consider a tax, still proportional to the value of the property, that the authority charges each time a unit of housing changes hands. We denote this by τ. It is of no economic consequence whether we assume this to be levied on the seller or the buyer, and we assume the former. As we will discuss equilibrium comparative statics as functions of tax rates, we now adopt notation, where, for example, $r^*(\tau)$ denotes the equilibrium rental price, given sales tax τ, but when it is unnecessary for clear exposition, we omit the argument.

Equilibrium. On the supply side, for endowed insiders to be indifferent between renting out and selling a given unit of housing, it must hold that

$$\lambda r^* = (1 - \tau) p^*. \tag{6}$$

On the demand side, the new wrinkle that arises under this regime is that unendowed and endowed insiders no longer face the same incentives as one another when choosing how much housing to consume. Let x_u^* and x_e^* denote their respective equilibrium consumption levels. As in Subsection 3.2, unendowed insiders housing consumption satisfies $u'(x_u^*) = p^*$. Endowed insiders, meanwhile, solve

$$\max_{x_e} \; u(x_e) + (1 - \tau) p^* \cdot (h_e - x_e),$$

implying that $u'(x_e^*) = (1 - \tau) p^*$, which reflects the fact that, for insiders, occupying an extra unit of housing brings about a lower opportunity cost than it does for outsiders. In order to occupy one more unit of housing, outsiders must pay to acquire it, thus sacrificing p^* yuan. In contrast, to enact the same increase in consumption, insiders must sell one fewer unit of their initial endowment. In doing so, they would sacrifice only the $(1 - \tau) p^*$ yuan that remain once the tax has been levied. For outsiders, the optimal rental quantity, as a function of r, remains unchanged. Therefore, in the presence of a sales tax and ownership restrictions on outsiders, the equilibrium allocation satisfies

$$u'(x_e^*) = (1 - \tau) u'(x_u^*) = \lambda \theta u'(y^*). \tag{7}$$

The Impact of a Small Sales Tax. We use this equilibrium characterization in order to derive Proposition 3.

Proposition 3. *Starting from $\tau = 0$, a marginal increase in the sales tax rate*

(a) leads to an increase in management frictions if and only if $MR(y^*) > 0$,

(b) brings about an increase in total utility from housing consumption,

(c) gives rise to an increase in total surplus.

To interpret this proposition, first note that, as a result of the sales tax, it becomes relatively more expensive for unendowed insiders to purchase housing. It also becomes relatively more desirable for endowed insiders to supply their excess endowment to the rental rather than sales market. As a result of these forces, there is a reduction in the inefficient gap, identified in Subsection 3.2, wherein outsiders rent too little and insiders own too much. However, another result of these forces is that endowed insiders increase their consumption. This not only pushes against a reduction in the gap between renters and owners, but also, it leads to a new inefficiency within the consumption profile of insiders, as the two subgroups no longer have equalized marginal utility. Part (b) establishes that, starting from a zero sales tax, the former, positive forces dominate, pushing total utility from consumption upward.

Regarding part (a), recall the discussion of marginal revenue and management frictions in Subsection 3.2. In view of that discussion and the fact that the increase in τ leads to an increase in outsiders' rental of housing, this result is straightforward. Finally, part (c) guarantees that the gain in utility, from the introduction of a small sales tax, exceeds the potential loss from increased management frictions. This implies that, provided the authority's objective function is concave, its optimal sales tax is positive.

The Optimal Sales Tax Under Isoelastic Utility. Proposition 4 states a closed-form solution to the optimal sales tax problem, under the assumption of Isoelastic utility.

Proposition 4. *Under the restricted ownership policy, when* $u(\cdot)$ *takes on the Isoelastic form specified in equation* (5)*, the total surplus-maximizing sales tax is*

$$\hat{\tau} = \frac{\eta (1-\lambda)}{\eta + \lambda - \eta\lambda + \lambda (\lambda\theta)^{-1/\eta} \frac{n_e}{n_o}} \in (0, 1-\lambda) . \tag{8}$$

To interpret this optimal tax rate, first consider the special case where $\eta = 1$ and thus $u(\cdot) = log(\cdot)$. Under this assumption, total revenue in the rental market remains constant regardless of the housing consumption profile, and, thus, any change in τ has no impact on total management frictions. Here, equation (8) simplifies to

$$\hat{\tau}|_{\eta=1} = \frac{n_o (1-\lambda)}{n_o + n_e/\theta}, \tag{9}$$

which increases with θ and decreases with λ.

To simplify further, consider the extreme case, in which $\theta = 1$, and there is thus no difference between the rental and ownership "consumption technologies." A first intuition, in this case, might be that the optimal tax should equalize the perceived rental and purchase prices, for buyers. This aim could be achieved by setting $\tau = 1 - \lambda$ (see equation (7)). However, such an impulse glosses over the issue, discussed above, that, in the presence of a sales tax, unendowed and endowed insiders perceive different opportunity costs of consuming one more unit of housing. Hence, if τ were set as high as $1 - \lambda$, although this would equalize the marginal utility from housing consumption of outsiders and unendowed insiders, it would lead to excessive consumption by endowed insiders. Instead, the pricing equation that prevails, with an optimal sales tax under these parameter values, address this dual tradeoff, satisfying

$$r^* = \frac{\frac{n_e}{\lambda} + n_o}{n_e + n_o} p^* > p^*.$$

Thus, more generally, the fact that, with constant management frictions, $\hat{\tau}$ increases with θ reflects the following idea. As the rental consumption technology gets better, the authority places more weight on the objective of equalizing outsiders' and unendowed insiders' consumption and less weight on the analogous objective between unendowed and endowed insiders.

Now consider the question of why the expression in (9) decreases with λ. First imagine what happens, in the absence of a sales tax, as λ approaches 1, meaning that the "management technology" becomes perfectly efficient. Here, the indifference condition, on the supply side, between renting out and selling, tends toward $r^* = p^*$. If this were the case, then, even under a policy of restricted ownership, equilibrium attains both the Second-Best and Consumption-Optimal benchmarks, without any further outside intervention.

However, as λ decreases, holding fixed $\tau = 0$, the gap between r^* and p^* gets larger. This leads endowed insiders to inefficiently "subsidize" sales over rental, in the manner discussed in Subsection 3.2. When the authority introduces a sales tax, it counteracts this widening gap between r^* and p^*, which reduces the inefficiency between the consumption levels of outsiders and unendowed insiders, at the cost of creating a gap between the latter group's consumption and that of endowed insiders.

The forces discussed in the preceding paragraphs remain at play when $\eta \neq 1$. However, in that more general environment, management frictions are no longer constant. Proposition 5 states the comparative statics in the more general environment.

Proposition 5. *Let $\hat{\tau}(\theta, \lambda, \eta)$ denote the optimal sales tax, as stated in equation (8), as a function of the rental consumption technology, θ, the management technology, λ, and the inverse elasticity of demand, η. It holds that*

(a) $\partial\hat{\tau}/\partial\theta > 0$ (the optimal sales tax increases as the rental consumption technology improves),

(b) $\partial\hat{\tau}/\partial\eta > 0$ (the optimal sales tax decreases as demand becomes more elastic),

(c) if $\eta \geq 1$, then $\partial\hat{\tau}/\partial\lambda < 0$, whereas, if $\eta < 1$, there exists a threshold management technology, $\widetilde{\lambda} \in (0,1)$, such that, if $\lambda < \widetilde{\lambda}$, then $\partial\hat{\tau}/\partial\lambda > 0$, and if $\lambda > \widetilde{\lambda}$, then $\partial\hat{\tau}/\partial\lambda < 0$.

When we include variable management frictions in the model, it becomes more challenging to precisely track the forces that drive the comparative statics. Nevertheless, parts (a) and (b) remain straightforward. Regarding a shift in θ, the tradeoff on the demand side discussed above remains dominant. Regarding a shift in η, the impact on management frictions appears to dominate. That is, as inverse elasticity of demand, η, increases, the marginal impact of rental on aggregate management frictions becomes smaller. This leaves the planner with a weaker incentive to discourage rental and, thus, it is optimal to increase the sales tax, τ. In case (c), regarding a shift in λ, the objectives concerning both marginal utility equalization and reduction of management frictions combine in a more complicated way. An increase in λ represents, on the one hand, a decrease in management frictions. On the other hand, this leads to an increase in rental supply. By itself, the first of these two effects would lead the authority to want to increase rental; however, the second effect, which leads to an endogenous increase rental, is strong enough to counteract this desire, except when both η and λ are sufficiently small.

4.3 Rental Tax

Now consider an alternative regime in which the authority taxes (or subsidizes) rental transactions. This rental tax instrument, denoted by σ, is analogous to the sales tax of Subsection 4.2 in that it specifies a proportion of the value of a rental transaction that the authority will collect. As before, the issue of whether this is levied on landlords or tenants has no economic impact, and we assume the former.

Equilibrium. On the supply side, the indifference condition between selling and renting out becomes

$$(\lambda - \sigma)\, r^* = p^*. \tag{10}$$

On the demand side, unlike with a sales tax, a rental tax does not drive a wedge between unendowed and endowed insiders' consumption. Both of these groups consumption satisfies $u'(x^*) = p^*$. Meanwhile, as before, outsiders' optimal rental level solves $\theta u'(y^*) = r$. Therefore, under ownership restrictions, with a rental tax, the equilibrium allocation satisfies

$$u'(x^*) = (\lambda - \sigma)\, \theta u'(y^*)\,. \tag{11}$$

Proposition 6 characterizes the impact of a small rental tax.

Proposition 6. *Starting from $\sigma = 0$, a marginal increase in the rental tax rate*

(a) leads to a decrease in management frictions if and only if $MR(y^*) > 0$,

(b) brings about a decrease in total utility from housing consumption,

(c) gives rise to a decrease in total surplus.

When rental transactions are taxed, this discourages rental, thereby widening the gap between insiders' and outsiders' marginal utilities from housing consumption. Moreover, unlike a sales tax, a rental tax does not create a wedge between the housing consumption levels of endowed versus unendowed insiders. This point leads to Proposition 7 regarding rental subsidy.

Proposition 7. *Let $\varepsilon_y^* \equiv -r^* / \left(y^* \frac{dr}{dy}\right)$ denote the elasticity of demand for rented housing at equilibrium under the restricted ownership policy, given σ. The Second-Best can be obtained by imposing a rental subsidy satisfying $\hat{\sigma} = -(1-\lambda)/\varepsilon_y^* < 0$.*

This formula for the optimal rental subsidy can be understood using the intuition developed in Subsection 3.2. In particular, note that, under log utility, the elasticity of rental demand is always equal to one. In that case, the optimal rental subsidy satisfies $\lambda - \hat{\sigma} = 1$. Plugging this into equation (11) gives $u'(x^*) = \theta u'(y^*)$. Meanwhile, when $\varepsilon_y^* \neq 1 \Leftrightarrow MR(y^*) \neq 0$, due to the marginal impact of rental on management frictions, the optimal rental subsidy leaves insiders and outsiders with different marginal utilities from housing consumption, as illustrated in Figure 1.

The result of Proposition 7, while perhaps elegant in theory, raises an important practical question. If the authority were to subsidize rental, although this would raise total surplus, doing so would also transfer wealth from unendowed insiders to both endowed insiders and outsiders. Recall, however, that, as we discuss at the end of Subsection 3.2, an effect of (and thus a potential motivation for) the restricted ownership policy, in the first place, is to make unendowed insiders better off, at the expense of the other two groups. Therefore, it is questionable whether a government whose interests lead it to impose an ownership restriction policy would also wish to subsidize rental. Consistent with this reasoning is the fact that, currently, the Beijing government taxes both sales and rental transactions at positive rates.

5 Extensions

5.1 Endogenous Migration

As discussed in the introduction, China's rural-to-urban migration is an important factor in the context of the housing issues that this paper studies. For simplicity, in the main model presented above, we hold fixed the number of both insiders and outsiders in the city. However, as we show in the more general model of Appendix B, all of our results continue to hold, subject to minor variation, when migration occurs endogenously. Here, we summarize the way in which we incorporate migration into the model and highlight the effect of doing so.

While still holding fixed the number of insiders, n_i, we assume that there are $\overline{n_o}$ *potential* outsiders. Each one makes a choice of whether to remain in his hometown or to migrate to the city. In making this decision, he takes into account (a) the rental price in the city, which determines his payoff from housing in the city, and (b) his own value of w, an idiosyncratic term that captures his net preference (positive or negative) for all other aspects of life in the city apart from housing – versus – in his hometown, his payoff from everything including housing. By assuming that different potential outsiders' values of w are continuously distributed, we can endogenously express the number of migrants as $n_o(y)$, where $n_o'(y) > 0$. Moreover, we can account in a straightforward way for migrants' welfare deriving from city life not related to housing.

The only aspect of the results stated above that migration changes is the appropriate notion of marginal revenue from rental. Recall that, in the main text's model, a small increase in y (housing consumption per outsider) leads to a change in management frictions that is proportional to $MR(y) = \frac{d}{dy}\{r(y)y\}$. With endogenous migration, a small change in y leads to a change in management frictions proportional to

$$\frac{d}{dy}\{n_o(y)r(y)y\} = (n_o(y) + n_o'(y)y)\, r + n_o(y)y\frac{dr}{dy},$$

which takes into account not only the intensive margin in the rental market, as before, but also the extensive margin of new migrants to the city. With endogenous migration, the main tradeoff – between, on the one hand, equalizing marginal utilities from housing consumption, and, on the other hand, minimizing management frictions – persists, and the welfare implications of the taxes we study are unchanged.

5.2 Dynamics

In this subsection we consider a two-period version of the model. This extension helps match the model to reality in two ways. The first is that it takes into account the fact that, in practice, outsiders who move to large Chinese cities can often, after fulfilling certain conditions over a period of time, obtain the right to purchase housing.[12] The second is that it generates a trajectory of equilibrium outcomes that can be compared to those reported in empirical studies. In particular, here, we adopt a parameter representing the strictness of the ownership restriction policy, as measured by the probability that a given outsider will retain this status in the second period, and we derive several positive predictions. These are: (a) the policy always causes a short-term drop in the housing sales price and the sales-to-rental price ratio; (b) the stronger the policy is, the larger and more permanent this drop in sales prices will be; (c) the weaker the policy is, the more investment demand there will be for housing. Finally, on the normative side, we show that, even when the policy is not strict and affects prices only temporarily, this still has a redistributive effect in favor of unendowed insiders.

[12] As mentioned in the introduction, the migrants in Beijing gain the ability to own housing, for example, after they obtain *hukou*, which can be applied for after they have paid local taxes for consecutive five years. Besides, according to the "grading" system recently established in Beijing and Shanghai, the non-local residents can be graded based on several aspects: e.g., identities, education achievements, career status, etc, and those who with relatively higher "scores" will be issued local *hukou*.

Specifically, in the extension, a share $\pi \in [0, 1]$ of the n_o agents who are outsiders in period 1 remain as outsiders in period 2. The other $1 - \pi$ share of outsiders from period 1 become unendowed insiders in period 2. Thus, we interpret π as a measure of the policy's durability or strictness. Preferences are the same as in the main model. In each period, insiders can buy/sell housing and occupy it, whereas outsiders may only rent. Agents who own housing at the end of the first period (which they had consumed and/or rented out) retain this as their period 2 endowment. In each period, there is a sales price p_t and a rental price r_t, and per-period demand levels are denoted analogously. Note, however, that for the purpose of measuring the sales price trajectory over time, the relevant series is $\left\{\frac{p_1}{2}, p_2\right\}$.[13] Here, we discuss the more novel issues mentioned above, formalizing them in Propositions 8 and 9. Appendix C contains all derivations related to this extension, showing that the main results from the baseline model all continue to go through.

Figure 2 displays the equilibrium housing prices over the periods as functions of the policy's strictness, π. The left panel also shows the equilibrium prices that arise in the absence of any ownership restrictions. Two points stand out. First, when the policy is strict, meaning outsiders durably retain this status, the policy permanently lowers prices. This outcome replicates, in a two-period setting, the results discussed in the main model. Interestingly, when the policy is not so strict, the housing price temporarily drops below its no-policy level before rising again. When $\pi = 0$, the period 2 price rises all the way up to the no-policy level.

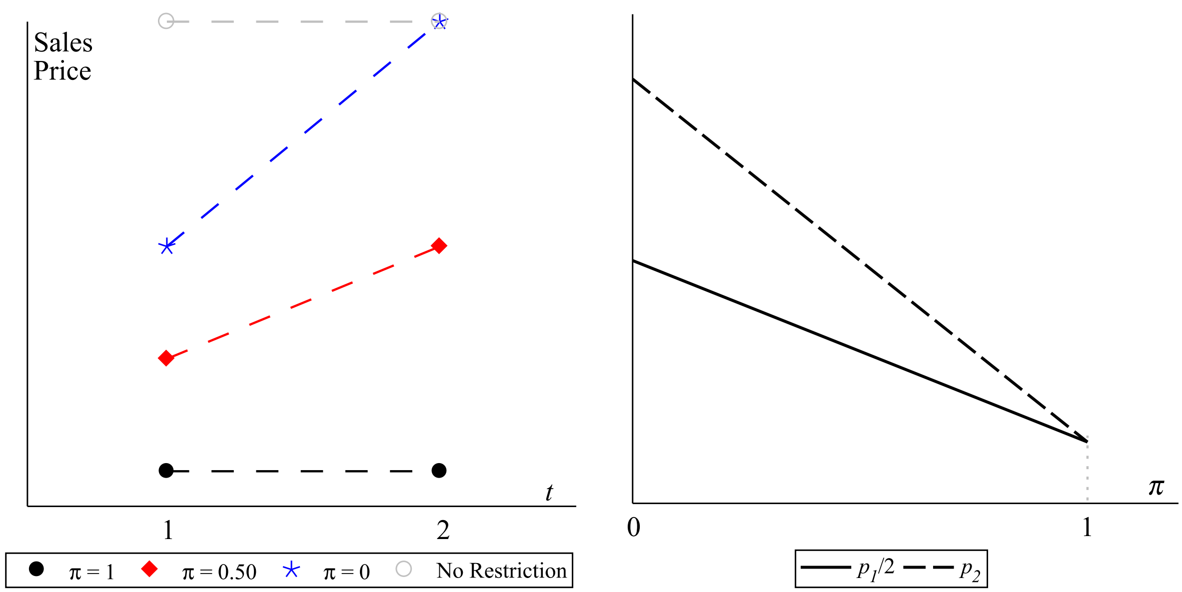

Figure 2: *The effect on the housing price trajectory of the policy's strictness level π.*

[13]This is because purchasing a unit of housing in period 1 effectively buys the ability to consume it or rent it out for twice as much time as purchasing a unit in period 2.

Proposition 8. *The first-period sales price is always lower with the ownership restriction policy than without it, regardless of the policy's strictness level, π. Morever, as π increases,*

(a) the first-period housing price and sales-to-rental price ratio decreases: $\frac{d}{d\pi}\left\{\frac{p_1^*}{2}\right\} < 0$, $\frac{d}{d\pi}\left\{\frac{p_1^*}{r_1^*}\right\} < 0$;

(b) the rise in the housing price between the two periods becomes smaller: $\frac{d}{d\pi}\left\{p_2^* - \frac{p_1^*}{2}\right\} < 0$.

(c) investment demand for housing decreases: $\frac{d}{d\pi}\left\{x_1^* - x_2^*\right\} < 0$.

For low values of π, the mechanism works as follows. The temporary stifling of outsiders' demand leads both endowed and unendowed insiders to consume more housing, in period 1, than they would have in the absence of the policy. This is what we refer to in part (c) as "investment demand," because, in temporarily consuming this housing, insiders anticipate its increased future value, planning to sell it or rent it out in the next period. The period 1 price incorporates this factor. As the policy becomes stricter, the potential diminishes to sell housing in the next period to new insiders. Therefore, a stricter policy leads to a lower sales price in both the first and second periods.[14] In light of these dynamics, Proposition 9 states the policy's effects on the groups' respective welfare.

Proposition 9. *(a) Following a change from no ownership restrictions to a policy with any level of $\pi \in [0, 1]$, or following an increase in π, unendowed insiders become better off and endowed insiders become worse off.*

(b) Under a restricted ownership policy, a given agent who is an outsider in period 1 who either (i) is certain to remain an outsider in period 2 or (ii) is certain to become an insider in period 2 benefits from an increase in π.

The logic behind part (b) of Proposition 9 is that, holding fixed the second-period status of the unendowed agent it is preferable for their to be fewer insiders who act as competition and drive up the housing price. In practice this result may be relevant, for example, from the perspective of highly educated migrants who expect no trouble in obtaining local *hukou* following a given waiting period. Although the restricted ownership policy temporarily harms them in early on, they benefit overall from the ability to obtain lower priced housing once they obtain insider status. At the same time, it may be relevant from the perspective of migrants with no qualifications to obtain local *hukou*, because they benefit more from having a larger fraction of the overall population persistently confined to renting. Despite this logic regarding

[14]Broadly speaking, these predictions appear consistent with empirical findings from Chinese cities while ownership restriction policies have been in place. For example, Cao et al. (2015) and Du and Zhang (2015) both find that cities with such policies indeed saw curbed sales prices for housing; studying Beijing, Sun et al. (2017) find similar results, post-adoption, as well as a drop in the sales-to-rental price ratio; in cities with such policies, Cao et al. (2018) report a positive correlation between current sales prices (which, in our model are inversely related to π) and investment demand.

an outsider whose second-period status is certain, we are not able to pin down the sign of the aggregate effect of an increase in π on outsiders' welfare. This is because, from the *ex ante* perspective of an outsider, when π increases, this increases the probability that she retains outsider status for both periods. Therefore, there is a tradeoff between, on the one hand, less competition bidding up the price of housing under a strict policy and, on the other hand, a higher chance to become an insider under a lax policy.

5.3 Wealth Effects

The main model's assumption of quasi-linear utility provides analytical convenience, but, as we show here, it does not drive our results. In this subsection, we explain intuitively why the basic distortion identified in the main model persists more generally. In Section 1 of the Online Appendix, we provide derivations, both general and for the example of Cobb-Douglas utility.

First, briefly consider a more general version of our setup. Each agent chooses a level of housing consumption (x if owned and y if rented) and a consumption level of a numéraire good, c, that need not enter the utility function in an affine manner, subject to a budget constraint. In particular, the budget constraint for an endowed insider is $\bar{c} + \max\{p, \lambda r\}(h_e - x) - c \geq 0$, where $\bar{c}$ is that agent's wealth, in units of the numéraire, which is separate from her endowment of housing, h_e. If $p > \lambda r$, then any housing supplied would be offered for sale; whereas, if $p < \lambda r$, then any housing supplied would be put up for rent. Therefore, under the restricted ownership policy, for both sales and rental markets to clear, $p = \lambda r$ must hold, as in the previous sections.

Next, let us compare this equilibrium, under the policy, with the one arising when there are no ownership restrictions (as in Benchmark A in Subsection 3.1). Given the basic assumption that housing is a normal good, it follows that $p^* < p^A < r$. Consequently, as in the quasi-linear version, the policy harms outsiders, by forcing them to rent and to do so at an unfavorable price. It also harms endowed insiders by decreasing the value of their housing assets. However, it helps unendowed insiders by decreasing the price they must pay to buy housing.

We can now examine why equilibrium under restricted ownership features the same kind of distortion, here, as arises with quasi-linear utility. Note that, unlike in that environment with transferable utility, we can no longer check for Pareto-Optimality simply by looking at the sum of total surplus, so, instead, we use an Edgeworth Box. Figure 3 depicts equilibrium under restricted ownership when agents have Cobb-Douglas preferences, and it shows the feasibility of a Pareto improvement. The horizontal axis represents the total housing consumed, at equilibrium, by a given endowed insider and a given outsider. The vertical axis represents the total amount consumed by these two agents, at equilibrium, of the numéraire. The endowed insiders' consumption set originates in the southwest, while that of the outsider originates in the northeast.

The two agents have budget lines of different slopes, which means that equilibrium is not

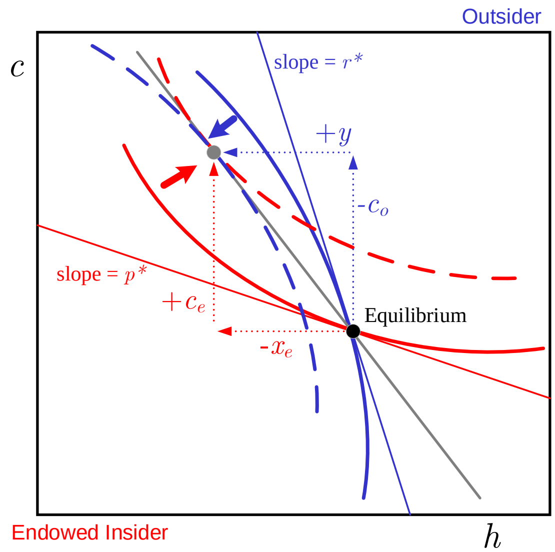

Figure 3: *An Edgeworth Box under Cobb-Douglas utility: starting from equilibrium, the arrows show the path to a Pareto Improvement for these two agents, holding fixed unendowed insiders' consumption (not pictured).*

a tangency point of their indifference curves (i.e., their marginal rates of substitution between housing and the numéraire are not equalized). Cobb-Douglas utility is analogous to the Isoelastic utility discussed in Subsection 3.2: agents to spend a constant share of their wealth on each good, and, therefore, total rental revenue and total management frictions are constant with respect to prices. Consequently, under Cobb-Douglas utility, Pareto optimality requires the allocation to lie at a tangency points between the agents' indifference curves. This can occur only if $p^B = r^B$. Under more general specifications, management frictions need not be constant, and so Pareto-Optima do not necessarily lie at such tangency points. Instead, as the constrained optimization exercise in the Online Appendix shows, Second-Best pricing must obey a rewritten version of equation (1),

$$\begin{aligned} p^B &= r - (1-\lambda)MR(y^B) \\ &= \lambda r^B - (1-\lambda)y^B \frac{dr^B}{dy} > \lambda r^B, \end{aligned}$$

where the "generalized" feature is that, now, p^B and r^B are not just partial derivatives of the respective agents' utility with respect to housing, but rather, marginal rates of substitution between housing and the numéraire.

The more general specification adopted in this subsection raises two additional points. First, the potential, emphasized in Section 4, for transaction-based taxes to improve welfare should be interpreted carefully. In practice, although such taxes or subsidies may increase an aggregate measure of welfare, they *alone* will not lead to Pareto improvements – a sales tax inevitably favors outsiders at the expense of insiders, while a rental subsidy favors endowed insiders and outsiders at the expense of unendowed insiders. As the current subsection illustrates, in order to lead to a Pareto improvement, such transactions taxes/subsidies would need to be part of a larger scheme involving transfers of wealth (here, the numéraire) from the subsidized group(s) to the harmed group(s). Within the context of the model, this suggests that there could be a role for property taxes, which, in principle, could be used in conjunction with rental subsidies in order to compensate unendowed insiders for the heightened competition in acquiring housing with the rental subsidy in place. In practice, however, without much deeper study, we view any such policy very skeptically, in part because it would seem be a "band-aid" whose best possible result would be to partially undo the distortion created, in the first place, by the ownership restriction policy.

Second, if transactions taxes were levied in a way that was not revenue-neutral, then their potential desirability depends on one's general view of the macroeconomic impact of government spending. This issue has long been the subject of debate in economics, and it is beyond this paper's scope to take a particular position. However, we may point out that, for someone of the view that the "multiplier" associated with government spending is high (meaning fiscal policy has a strong potential to stimulate the economy), a sales tax would be relatively more desirable than it would be for someone who believes this number to be low. This is because a sales tax raises a positive level of revenue that the government could then spend on other projects such as infrastructure, services, or other public goods. Regarding rental subsidies, if (in a manner beyond what we explicitly consider in the model) one were to hold fixed the government's tax receipts, then the mapping between these two positions becomes reversed. That is, the optimal subsidy level is a decreasing function of the multiplier, because the opportunity cost of money spent on rental subsidies increases with the multiplier. For more on this issue of the macroeconomic effects of government spending, see, for instance, Burdekin and Weidenmier (2015) and Wang and Wen (2013), focusing on China, and, more recently, Ramey and Zubairy (2018) regarding the U.S.

5.4 Heterogeneity

In the main model, we assume that all outsiders have the same value of θ, the coefficient measuring their distaste for renting rather than owning their house. We also assume that all endowed insiders begin with the same amount of housing, h_e. Section 2 of the Online Appendix relaxes both of these assumptions. In it, each outsider's θ is drawn from a continuous distribution with support over interval $[\underline{\theta}, \overline{\theta}]$, and each insider's endowment is drawn from a continuous distribution with support over interval $[0, \overline{h}]$.[15]

1. The relevant expressions for marginal revenue in the rental market, which appear in Propositions 2, 3, 6, and 7 are integrals over the distribution of θ.

2. Among insiders, the salient distinction ceases to be the binary one between "unendowed" and "endowed." Instead, there are two relevant thresholds to consider: One is the first-best equilibrium consumption level, x^A, and the other is insiders' equilibrium consumption level, x^*. Recall that $x^A < x^*$. When the restricted ownership policy is introduced, its effect on the welfare of insiders with endowments less than x^A is unambiguously positive in a manner analogous to its effect on the unendowed in the binary model. This is because, under both regimes, they are net demanders of housing. The policy's effect on the welfare of insiders with endowments greater than x^* is unambiguously negative in a manner analogous to its effect on the endowed in the binary model. Under the ownership restriction policy, compared to the first-best, insiders with endowments in between these two levels switch from being net suppliers to net demanders, so the effect on this group is ambiguous. However, it is straightforward to see that a single threshold exists, between x^A and x^*, that separates insiders who are helped by the policy from those who are harmed.

Finally, consider potential heterogeneity in λ, the parameter associated with management frictions. As this is a bit messier to incorporate into the model, we, instead, discuss it informally. First, make the straightforward assumption that some endowed insiders are more skilled/effective than others as potential landlords. Then, there is an idiosyncratic distribution of λ. In such a scenario, at equilibrium there would be a threshold value, $\overline{\lambda}$, satisfying $p^* = \overline{\lambda} r^*$, and suppliers with λ below this threshold would be sellers, whereas those above it would be landlords. If things were assumed to work this way, then the results of the model go through in much the same way as with a heterogenous distribution of θ.

However, note that such an outcome appears unstable. To see why, consider two candidate landlords, 1 and 2, with $\lambda_1 > \lambda_2 > \overline{\lambda}$. There would be an incentive for these two agents to come to an arrangement in which 1 purchases 2's property at some price $\widehat{p} \in (\lambda_2 r^*, \lambda_1 r^*)$ and then uses her superior abilities to rent out the property. This argument, however, taken to its logical end would lead to the absurd-seeming conclusion that there would be only one landlord in the entire city, i.e., the agent with the highest λ. A more reasonable interpretation of λ, therefore, may be as a technology parameter capturing "best practices" for renting out housing that, to a first approximation, are shared among insiders who choose to become landlords. Of course,

[15]In reality, there are also cases of non-residents who already owned housing before the policy came into effect. Our understanding is that policies allow such outsiders to maintain ownership of their property but not to purchase more. Thus, such people are approximately like endowed insiders. The one exception, however, which we ignore in this extension, are outsiders whose existing endowment is small enough so that they would still be net demanders.

in reality, there are both large companies that benefit from economies of scale in renting out property and, at the same time, individual landlords who are likely less efficient, suggesting that some combination of these two extreme scenarios would be a more accurate description of what actually occurs.[16]

6 Conclusion

In this paper, we explore the economic impact of a housing policy that has been in place major Chinese cities, since 2010, dictating who can own housing. This policy allows some residents, "insiders," to own housing, while restricting other residents, "outsiders," from doing so, thus forcing them to resort to the rental market in order to procure lodging. To examine a set of issues raised by such a policy, we build simple, tractable model whose results we find to be robust, via a series of generalizations and extensions.

We show that when outsiders are prohibited from purchasing housing, competitive equilibrium fails to attain a "second-best" efficient allocation that would maximize welfare subject to the constraint dictated by the policy. We further point out that the main beneficiaries of such a policy are the "unendowed" insiders, i.e., residents who have the right to purchase housing but who do not initially own any. On the other hand, both "endowed" insiders, who do initially own housing, and outsiders are harmed.

Then, in light of current debate over tax reform in the Chinese real estate market, we study the effects of several forms of taxation. Whereas property taxes simply extract wealth from initial owners of housing, transaction taxes can have subtler, potentially positive effects. In particular, under a regime with this kind restricted housing ownership, a small, positive sales tax always improves welfare compared to no tax at all. This is because such a tax helps correct the tendency for rental prices to be too high. At the same time, however, the imposition of such a tax gives rise to a distortion that would otherwise be absent, namely that endowed insiders are incentivized to over-consume housing relative to unendowed insiders. An optimal sales tax is positive, balancing these two effects.

Meanwhile, a subsidy on rental transactions can restore the second-best. However, in practice, a rental subsidy is likely undesirable from the standpoint of a government that imposes a restricted ownership restriction. This is because a rental subsidy favors endowed insiders and outsiders, at the expense of unendowed insiders. Thus, it would counteract the distributional effects that likely motivate the restricted ownership policy in the first place. This reasoning regarding both types of transaction taxes appears consistent with the current practice

[16] For example, a recent report (Xinhua and Ziroom, 2019) claims that, increasingly, rather than renting out property by themselves, property owners in China who wish to supply to the rental market use rental platforms to manage their properties. It further claims that the average length of time during which a platform manages a given property has risen from 3.2 to 5.1 years.

in Beijing, where both sales and rental housing transactions are taxed at a positive rate.

It goes without saying that the current model captures only a tiny part of the overall picture of the Chinese housing market. In particular, the statements we make about the welfare effects of different forms of taxation should be clearly understood to apply strictly within our framework and not as recommendations for any particular policies. Nevertheless, we believe that the mechanisms identified in this paper can be helpful both in helping to guide further empirical study and for thinking through the impact of proposed policies in this area.

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Appendices

A Proofs of Results from the Main Model

Proof of Lemma 1. First note that $MR(y) = \theta u'(y) + y\theta u''(y)$. This is implied by the constraint, stemming from outsiders' utility maximization, that $\theta u'(y) = r$, which, itself, implies $\theta u''(y) = dr/dy$. These results can then be obtained directly by substituting $MR\left(y^B\right)$ into the last term of equation (1) and comparing to equation (2). □

Proof of Proposition 1. $y^C \leq x^C$ follows from equation (2), which implies that $u'\left(x^C\right) \leq u'\left(y^C\right)$. The inequalities $y^* < y^C$ and $x^C < x^*$ follow from the fact that, to support demand profile $\left(x^C, y^C\right)$ would require $p = r$, but $p^* < r^*$, and aggregate housing consumption remains fixed across the two regimes. □

Proof of Proposition 2. $y^* < y^B$ and $x^B < x^*$, because, under the Second-Best, equation (1) implies that $p^B = r^B - (1-\lambda) MR\left(y^B\right) = \lambda r^B - (1-\lambda) y^B \theta u''\left(y^B\right) > \lambda r^B$, but $p^* = \lambda r^*$, and aggregate demand for housing remains fixed across the two regimes. □

Proof of Claim 1. Let ϕ denote the property tax. On the supply side. Given prices p and r, an endowed insider who chooses to retain ownership of the initial h_e units, while consuming only quantity x, pays property tax on the entire endowment and receives a payoff, net of outside wealth, of

$$u(x) + \lambda r \cdot (h_e - x) - \phi p h_e. \tag{A.1}$$

Meanwhile, such an agent who sells all but x units of the initial endowment receives payoff

$$u(x) + p \cdot \left(h_e - \left(1+\phi\right) x\right). \tag{A.2}$$

Thus, in order for there to be positive supply in both the sales and rental markets, expressions (A.1) and (A.2) must be equalized, which implies that

$$\lambda r^\phi = \left(1+\phi\right) p^\phi. \tag{A.3}$$

On the demand side, insiders and outsiders maximize utility by choosing quantities x^ϕ and y^ϕ, respectively, that satisfy

$$u'\left(x^\phi\right) = \left(1+\phi\right) p \quad \text{and} \quad \theta u'\left(y^\phi\right) = r^\phi. \tag{A.4}$$

The supply equation in (A.3) and the demand equations in (A.4), together, imply that $u'\left(x^\phi\right) = \lambda\theta u'\left(y^\phi\right)$, which, in turn, implies that $\left(x^\phi, y^\phi\right) = (x^*, y^*)$.

To see the impact on the different groups' welfare, first note that $y^{\phi} = y^* \Leftrightarrow r^{\phi} = r^*$, and, thus, outsiders perceive no impact from the tax, as they participate only on the demand side of the rental market. Furthermore, unendowed insiders are neither harmed nor helped: $x^{\phi} = x^* \Leftrightarrow \left(1+\phi\right) p^{\phi} = p^*$, which means that, when there is a positive property tax, the obligation to pay this that "comes with the property" is perfectly offset by a reduction in the purchase price. Therefore, endowed insiders bear the entire burden of the property tax, but such a tax is merely a transfer from endowed insiders to the authority with no effect on total surplus. □

Proof of Proposition 3. At equilibrium, with sales tax τ, total surplus is

$$\underbrace{n_e u\left(x_e^*(\tau)\right) + n_u u\left(x_u^*(\tau)\right) + n_o \theta u\left(y^*(\tau)\right)}_{\text{utility from housing}} - \underbrace{(1-\lambda)\, n_o y^*(\tau)\, \theta u'\left(y^*(\tau)\right)}_{\text{management frictions}}. \tag{A.5}$$

Total differentiation of equation (7), using the constraint that $n_e x_e^* + n_u x_u^* + n_o y^* = H \Rightarrow n_e \frac{dx_e^*}{d\tau} + n_u \frac{dx_u^*}{d\tau} + n_o \frac{dy^*}{d\tau} = 0$, gives

$$\frac{dy^*}{d\tau} = \frac{-n_u u'\left(x_u^*\right) u''\left(x_e^*\right)}{n_e \lambda\theta (1-\tau)\, u''\left(x_u^*\right) u''\left(y^*\right) + n_u \lambda\theta u''\left(x_e^*\right) u''\left(y^*\right) + n_o (1-\tau)\, u''\left(x_e^*\right) u''\left(x_u^*\right)} > 0. \tag{A.6}$$

For part (a), the sign of equation (A.6), combined with the result of Lemma 1, is sufficient. For part (b), using (7) and $n_e \frac{dx_e^*}{d\tau} + n_u \frac{dx_u^*}{d\tau} = -n_o \frac{dy^*}{d\tau}$, the derivative of the first term in equation (A.5), evaluated at $\tau = 0$, is

$$(1-\lambda)\, n_o \theta u'\left(y^*\right) \frac{dy^*}{d\tau} > 0.$$

For part (c), the derivative of equation (A.5), evaluated at $\tau = 0$, simplifies to

$$-(1-\lambda)\, n_o y^* \theta u''\left(y^*\right) \frac{dy^*}{d\tau} > 0. \tag{A.7}$$

□

Proof of Proposition 4. Under isoelastic utility, as defined in (5), $u'(x) = x^{-\eta}$, when $\eta \neq 1$, and $u'(x) = 1/x$, when $\eta = 1$. The equilibrium consumption levels can be solved by plugging the isoelastic utilities into the equilibrium conditions stated in (7). Denote total surplus, as expressed in (A.5), by $W(\tau)$, and the optimal sales tax, $\hat{\tau}$, solves $\frac{dW(\tau)}{d\tau} = 0$. See Online Appendix for detailed derivations. □

Proof of Proposition 5. Equation (8) states the optimal sales tax, $\hat{\tau}\left(\theta, \lambda, \eta\right)$. For part (a),

$$\frac{\partial \hat{\tau}}{\partial \theta} = \frac{(1-\lambda) n_e \lambda \left(\lambda\theta\right)^{-1/\eta}}{\left(\lambda + \eta(1-\lambda) + \lambda\left(\lambda\theta\right)^{-1/\eta} \frac{n_e}{n_o}\right)^2 \theta n_o} > 0.$$

For part (b),

$$\frac{\partial \hat{\tau}}{\partial \eta} = \frac{(1-\lambda)n_o\lambda\left(n_e\left(\eta - \ln(\lambda\theta)\right)(\lambda\theta)^{-1/\eta} + \eta n_o\right)}{\eta\left(-n_e\lambda\left(\lambda\theta\right)^{-1/\eta} - \left(\eta(1-\lambda)+\lambda\right)n_o\right)^2} > 0.$$

For part (c), $\partial\hat{\tau}/\partial\lambda$ has a square term as its denominator, and, thus, its sign is the same as that of its numerator, which is

$$-\left(n_e\left(\eta-(1-\lambda)\right)(\lambda\theta)^{-1/\eta}+\eta n_o\right)n_o. \tag{A.8}$$

When $\eta \geq 1$, the expression in (A.8) is negative. When $\eta \in (0,1)$, the expression in (A.8) is positive when evaluated at some positive value of λ sufficiently close to zero, it is negative when evaluated at $\lambda = 1$, and, over the interval $\lambda \in (0,1)$, it is continuously decreasing in λ. □

Proof of Proposition 6 . Given rental tax σ, total surplus is given by equation

$$\underbrace{n_i u\left(x^*(\sigma)\right) + n_o\theta u\left(y^*(\sigma)\right)}_{\text{utility from housing}} - \underbrace{(1-\lambda)n_o\theta u'\left(y^*(\sigma)\right)y^*\left(\sigma\right)}_{\text{management frictions}}. \tag{A.9}$$

Total differentiation of equation (11), using the constraint that $n_i x^* + n_o y^* = H \Rightarrow n_i\frac{dx^*}{d\sigma} + n_o\frac{dy^*}{d\sigma} = 0$, gives

$$\frac{dy^*}{d\sigma} = \frac{n_o\theta u'(y^*)}{n_o u''(x^*) + n_i(\lambda-\sigma)\theta u''(y^*)} < 0. \tag{A.10}$$

The derivative of the second term of (A.9), evaluated at $\sigma = 0$, is

$$(1-\lambda)n_o\theta\left(u''(y^*)y^* + u'(y^*)\right)\frac{dy^*}{d\sigma}. \tag{A.11}$$

For part a, the sign of (A.10), combined with the result of Lemma 1, is sufficient. For part b, using (11) and $n_i\frac{dx^*}{d\sigma} = -n_o\frac{dy^*}{d\sigma}$, the derivative of the first term of equation (A.9) , evaluated at $\sigma = 0$, simplifies to

$$n_o(1-\lambda)\theta u'(y^*)\frac{dy^*}{d\sigma} < 0.$$

For part c, using (A.10), the derivative of equation (A.9), evaluated at $\sigma = 0$, simplifies to

$$-n_o(1-\lambda)\theta u''(y^*)y^*\frac{dy^*}{d\sigma} < 0.$$

□

Proof of Proposition 7. Denote total surplus, as expressed in (A.9), by $W(\sigma)$. Its derivative is

$$\frac{dW(\sigma)}{d\sigma} = n_i u'(x^*)\frac{dx^*}{d\sigma} + n_o\theta u'(y^*)\frac{dy^*}{d\sigma} - (1-\lambda)n_o\theta\left(u''(y^*)y^* + u'(y^*)\right)\frac{dy^*}{d\sigma}. \tag{A.12}$$

Using (11) and $n_i\frac{dx^*}{d\sigma} = -n_o\frac{dy^*}{d\sigma}$, (A.12) is simplified to

$$\left(u'(y^*)\sigma - (1-\lambda)u''(y^*)y^*\right)n_o\theta\frac{dy^*}{d\sigma}. \tag{A.13}$$

The optimal rental tax,

$$\hat{\sigma} = (1-\lambda)\frac{y''(y^*)y^*}{u'(y^*)},$$

solves $\frac{dW(\sigma)}{d\sigma} = 0$. The elasticity of demand for rented housing at equilibrium, is given by

$$\varepsilon_y^* = -\frac{u'(y^*)}{u''(y^*)y^*}.$$

□

B Endogenous Migration and Housing Supply with Generalized Management Frictions

We generalize the main model by allowing for endogenous migration and housing supply as well as per-unit management frictions. We show that all of the general properties of the baseline model extend naturally. Formally, here we introduce the following modifications.

1. In addition to the n_i insiders, there are $\overline{n_o}$ *potential* outsiders each of whom may choose to move to the city. Aside from the net utility they derive from housing in the city, outsiders receive a payoff w from moving to the city, representing their net preference (either positive or negative) for all other aspects of life in the city versus their hometowns. w is heterogeneously distributed according to CDF $F(\cdot)$ and PDF $f(\cdot)$, both assumed to be continuous. A given potential outsider moves to the city if and only if $\theta u(y) - \theta u'(y)y + w \geq 0$, where the right-hand side represents the normalized payoff from remaining in one's hometown. The total number of outsiders who choose to live in the city is then

   $$n_o(y) = \overline{n_o}\int_{-\theta u(y)+\theta u'(y)y}^{+\infty} dF(w).^{17}$$

   Note that $n_o'(y) = -\theta u''(y)yf\left(-\theta u(y) + \theta u'(y)y\right)\overline{n_o} > 0.$

[17] $n_o(y)$ may be interpreted as the expected number of outsiders, where we ignore the integer constraint that would affect the realized number, or, alternatively, the economy may be assumed to be continuous.

2. Besides housing initially owned by endowed insiders, $H = n_e h_e$, there is a production sector which could supply additional housing H_s, so the total supply is $\widehat{H} = H + H_s$. Here, we assume the production sector is perfectly competitive and the production technology is characterized by increasing, convex total costs, $C(H_s)$, giving rise to zero long-run profits. Thus aggregate supply $S(p) = MC^{-1}(p)$.[18]

3. For each unit rented out, a landlord incurs constant cost $\kappa > 0$, in addition to the proportional cost $(1-\lambda)r$.[19]

Except where otherwise indicated, all results stated in this section follow from the same arguments used in Appendix A.

B.1 The Impact of Ownership Restriction

The "Second-Best" benchmark now solves $\max_{x,y}\ n_i u(x) + n_o \theta u(y) - C(n_i x + n_o y - H) - n_o\left((1-\lambda)r + \kappa\right)y + \int_{-\theta u(y)+\theta u'(y)y}^{+\infty} w f(w) dw$, subject to $\theta u'(y) = r$. The solution is

$$u'(x^B) = \lambda\theta u'(y^B) - \kappa - \frac{(1-\lambda)n_o^B \theta u''(y^B) y^B}{n_o^B + n_o^{B\prime} y^B} = MC(n_i x^B + n_o^B y^B - H). \tag{B.1}$$

"Consumption-Optimal" solves $\max_{x,y}\ n_i u(x) + n_o \theta u(y) - C(n_i x + n_o y - H)$, which yields

$$u'(x^C) = \frac{n_o^C + n_o^{C\prime} y^C \frac{u(y^C)}{u'(y^C) y^C}}{n_o + n_o^{C\prime} y^C}\theta u'(y^C) = MC(n_i x^C + n_o^C y^C - H). \tag{B.2}$$

Recall, $MR(y) \equiv r + y\frac{dr}{dy}$ denotes the *Marginal Revenue* function associated with the rental demand of an outsider living in the city. When there is endogenous migration, a key difference, compared to the model in the main text, is that the rental market has not only an intensive margin but also an extensive one. To account for this aspect, it is convenient to introduce notation for the *intensive marginal share*, $m(y) \equiv \frac{n_o(y)}{n_o(y)+n_o'(y)y} \in [0,1]$. When per-outsider housing consumption, y, increases by a small amount, $m(y)$ denotes the fraction of additional total outsider housing consumed by "infra-marginal" outsiders, i.e., those who receive strictly positive net payoffs from moving to the city and thus merely increase their consumption level. The complementary share, $1 - m(y) = \frac{n_o'(y)y}{n_o(y)+n_o'(y)y}$, denotes the *extensive marginal share*, consumed by "new" (i.e., marginal) migrants who are barely drawn to the city by the rental price decrease

[18] We make the assumption of perfect competitive production in order to be consistent with our welfare analysis in the baseline model. Without this assumption, supply $S(p)$ can be interpreted to be any supply curve as a function of price. When production is not perfectly competitive, however, deadweight loss will occur, due to firms' market power.

[19] We assume that λ and κ are, respectively, large and small enough so that the rental market is active.

and thus go from consuming 0 to consuming y units. For simplicity, we often refer to $m(y^*)$ as m^* and so forth.

We now state Lemma 1′, which is analogous, in this model to Lemma 1 in the main text, and we follow this labeling convention throughout Appendix B.

Lemma 1′. *When* $MR\left(y^B\right) > -\frac{\kappa+(1-m^B)\left(\frac{\theta u(y^B)}{y^B}-\lambda r^B\right)}{m^B(1-\lambda)}$*, the authority favors sales over rental, under the Second-Best regime, compared to the outcome under the Consumption-Optimal regime. When* $MR\left(y^B\right) < -\frac{\kappa+(1-m^B)\left(\frac{\theta u(y^B)}{y^B}-\lambda r^B\right)}{m^B(1-\lambda)}$*, the opposite is true, and, when* $MR\left(y^B\right) = -\frac{\kappa+(1-m^B)\left(\frac{\theta u(y^B)}{y^B}-\lambda r^B\right)}{m^B(1-\lambda)}$*, the allocations under the two regimes are identical. Formally,*

$$\begin{aligned}\operatorname{sign}\left\{u'(x^C)-u'(x^B)\right\} &= \operatorname{sign}\left\{x^B-x^C\right\} = \operatorname{sign}\left\{y^C-y^B\right\}\\ &= \operatorname{sign}\left\{MR\left(y^B\right)+\frac{\kappa+(1-m^B)\left(\frac{\theta u(y^B)}{y^B}-\lambda r^B\right)}{m^B(1-\lambda)}\right\}.\end{aligned}$$

Remark. $MR(y^B) \geq 0$ is sufficient for the first condition stated in Lemma 1′ to hold, since $-\frac{\kappa+(1-m^B)\left(\frac{\theta u(y^B)}{y^B}-\lambda r^B\right)}{m^B(1-\lambda)} < 0$.

Equilibrium Under Restricted Ownership. At equilibrium $u'(x^*) = p^*, \theta u'(y^*) = r^*, p^* = \lambda r^* - \kappa$. To close the model, the market clearing condition is $n_i x^* + n_o^* y^* = H + S(p^*)$. The equilibrium admits

$$u'(x^*) = \lambda\theta u'(y^*) - \kappa = MC(n_i x^* + n_o^* y^* - H). \tag{B.3}$$

Proposition 1′. *Compared to benchmark C, which maximizes total utility from housing consumption, equilibrium allocates less housing to rental and more to ownership. Moreover, these outcomes satisfy*

$$y^* < y^C,\ x^C < x^*. \tag{B.4}$$

Proposition 2′. *Compared to benchmark B, which maximizes total surplus, equilibrium allocates less housing to rental and more to ownership. That is,* $y^* < y^B$ *and* $x^B < x^*$*, but a complete ranking cannot be guaranteed.*

B.2 The Effects of Taxation Under Ownership Restriction

B.2.1 Property Tax

The proof of Claim 1 shows that, holding fixed the aggregate supply of housing, equilibrium consumption levels x^* and y^* are independent of the level of property tax, ϕ. Therefore, the only channel through which ϕ affects total surplus is its effect on aggregate supply of housing. As this supply decreases, so do x^* and y^*. It is thus straightforward to show that, in the current setting, the following claim is true.

Claim 1′. *Starting from $\phi = 0$, a marginal increase in the property tax rate leads to*

(a) a decrease in management frictions if and only if $MR(y^*) = r^* + y^* \frac{dr}{dy}\Big|_{y=y^*} > -\frac{\kappa+(1-m^*)(1-\lambda)r^*}{(1-\lambda)m^*}$,

(b) a decrease in total utility from housing consumption (net of production costs), and

(c) a decrease in total surplus.

B.2.2 Sales Tax

The equilibrium under tax rate τ features

$$u'(x_e^*) = (1-\tau)p^*, u'(x_u^*) = p^*, \theta u'(y^*) = r^*, (1-\tau)p^* = \lambda r^* - \kappa, n_e x_e^* + n_u x_u^* + n_o^* y^* = H + S\Big((1-\tau)p^*\Big). \tag{B.5}$$

Total surplus is

$$\underbrace{n_e u\left(x_e^*(\tau)\right) + n_u u(x_u^*(\tau)) + n_o\left(y^*(\tau)\right)\theta u\left(y^*(\tau)\right) - C\left(n_e x_e^*(\tau) + n_u x_u^*(\tau) + n_o\left(y^*(\tau)\right) y^*(\tau) - H\right)}_{\text{utility from housing (net of production costs)}}$$
$$-\underbrace{n_o\left(y^*(\tau)\right)\left[(1-\lambda)\theta u'\left(y^*(\tau)\right) + \kappa\right] y^*(\tau)}_{\text{management frictions}} + \underbrace{\overline{n_o}\int_{-\theta u(y^*(\tau))+\theta u'(y^*(\tau))y^*(\tau)}^{+\infty} w f(w)dw}_{\text{outsiders' aggregate surplus from city life}}. \tag{B.6}$$

Proposition 3′. *Starting from $\tau = 0$, a marginal increase in the sales tax rate*

(a) leads to an increase in management frictions if and only if $MR(y^*) = r^* + y^* \frac{dr}{dy}\Big|_{y=y^*} > -\frac{\kappa+(1-m^*)(1-\lambda)r^*}{(1-\lambda)m^*}$,

(b) brings about an increase in total utility from housing consumption (net of production costs),

(c) gives rise to an increase in total surplus.

Proof. Total differentiating equation (B.5) gives

$$\begin{aligned}\frac{dy^*}{d\tau} =& -n_u u'(x_u^*)u''(x_e^*)\Big[n_e\lambda\theta(1-\tau)u''(x_u^*)u''(y^*) + n_u\lambda\theta u''(x_e^*)u''(y^*) \\ &+ \left(n_o^* + n_o^{*\prime} y^*\right)(1-\tau)u''(x_e^*)u''(x_u^*) - \lambda\theta(1-\tau)u''(x_e^*)u''(x_u^*)u''(y^*)S'\left(u'(x_e^*)\right)\Big]^{-1} > 0.\end{aligned} \tag{B.7}$$

For part (a), the derivative of the second term of (B.6), evaluated at $\tau = 0$, is

$$(n_o^* + n_o^{*\prime} y^*) \left[(1-\lambda)\left(m^* MR(y^*) + (1-m^*)\theta u'(y^*)\right) + \kappa\right] \frac{dy^*}{d\tau}.$$

For part (b), the derivative of the first term of (B.6), evaluated at $\tau = 0$, is

$$(n_o^* + n_o^{*\prime} y^*) \left[m^*(1-\lambda)\theta u'(y^*) + (1-m^*)\left(\frac{\theta u(y^*)}{y^*} - \lambda r^*\right) + \kappa \right] \frac{dy^*}{d\tau} > 0.$$

For part (c), combining the derivative of the last term of equation (B.6), i.e., $-n_o^{*\prime}\left[\theta u(y^*) - \theta u'(y^*) y^*\right] \frac{dy^*}{d\tau}$, the derivative of equation (B.6), evaluated at $\tau = 0$, simplifies to

$$-n_o^*(1-\lambda)\theta u''(y^*) y^* \frac{dy^*}{d\tau} > 0.$$

□

B.2.3 Rental Tax

The equilibrium under tax rate σ features

$$u'(x^*) = p^*, \theta u'(y^*) = r^*, p^* = \left(\lambda - \sigma\right) r^* - \kappa, n_i x^* + n_o^* y^* = H + S\left(p^*\right). \tag{B.8}$$

Total surplus is

$$\begin{aligned} &\underbrace{n_i u\left(x^*(\sigma)\right) + n_o\left(y^*(\sigma)\right)\theta u\left(y^*(\sigma)\right) - C\left(n_i x^*(\sigma) + n_o\left(y^*(\sigma)\right) y^*(\sigma) - H\right)}_{\text{utility from housing (net of production costs)}} \\ &- \underbrace{n_o(y^*(\sigma))\left[(1-\lambda)\theta u'(y^*(\sigma)) + \kappa\right] y^*(\sigma)}_{\text{management frictions}} + \underbrace{\overline{n_o} \int_{-\theta u(y^*(\sigma)) + \theta u'(y^*(\sigma)) y^*(\sigma)}^{+\infty} w f(w) dw.}_{\text{outsiders' aggregate surplus from city life}} \end{aligned} \tag{B.9}$$

Proposition 6′. *Starting from $\sigma = 0$, a marginal increase in the rental tax rate*

(a) leads to a decrease in management frictions if and only if $MR(y^*) > -\frac{\kappa + (1-m^*)(1-\lambda) r^*}{(1-\lambda) m^*}$,

(b) brings about a decrease in total utility from housing consumption (net of production costs),

(c) gives rise to a decrease in total surplus.

Proof. Totally differentiating equation (B.8) gives

$$\frac{dy^*}{d\sigma} = \frac{\theta u'(y^*)\left(n_i - u''(x^*) S'\left(u'(x^*)\right)\right)}{(n_o^* + n_o^{*\prime} y^*)\, u''(x^*) + \theta u''(y^*)(\lambda - \sigma)\left(n_i - u''(x^*) S'\left(u'(x^*)\right)\right)} < 0 \tag{B.10}$$

For part (a), the derivative of the second term of (B.9), evaluated at $\sigma = 0$, is

$$(n_o^* + n_o^{*\prime} y^*) \left[(1-\lambda)\left(m^* MR(y^*) + (1-m^*)\theta u'(y^*)\right) + \kappa\right] \frac{dy^*}{d\sigma}.$$

For part (b), the derivative of the first term of (B.9), evaluated at $\sigma = 0$, is

$$(n_o^* + n_o^{*\prime} y^*) \left[m^*(1-\lambda)\theta u'(y^*) + (1-m^*)\left(\frac{\theta u(y^*)}{y^*} - \lambda r^*\right) + \kappa\right] \frac{dy^*}{d\sigma} < 0.$$

For part (c), combining the derivative of the last term of equation (B.9), i.e., $-n_o^{*\prime}\left[\theta u(y^*) - \theta u'(y^*)y^*\right] \frac{dy^*}{d\sigma}$, the derivative of equation (B.9), evaluated at $\sigma = 0$, simplifies to

$$-n_o^*(1-\lambda)\theta u''(y^*) y^* \frac{dy^*}{d\sigma} < 0.$$

□

Proposition 7′. *The Second-Best can be obtained by imposing a rental subsidy satisfying* $\hat{\sigma} = -(1-\lambda) m^* / \varepsilon_y^* < 0$.

Proof. Denote total surplus, as expressed in (B.9), by $W(\sigma)$. Its derivative simplifies to

$$\left[(n_o(y^*) + n_o'(y^*)y^*)\, \sigma \theta u'(y^*) - (1-\lambda) n_o(y^*) \theta u''(y^*) y^*\right] \frac{dy^*}{d\sigma}.$$

The optimal rental tax,

$$\hat{\sigma} = (1-\lambda) m^* \frac{\theta u''(y^*) y^*}{\theta u'(y^*)},$$

solves $\frac{dW(\sigma)}{d\sigma} = 0$. Evaluated at $\hat{\sigma}$, $u'(x^*) = \lambda r^* - \kappa - (1-\lambda) m^* \theta u''(y^*) y^*$, which is equivalent to (B.1). □

C A Two-Period Model

Here we present a two-period extension of the main model. In it, a share $\pi \in [0, 1]$ of the n_o agents who are outsiders in period 1 remain as outsiders in period 2. The other $1 - \pi$ share of outsiders from period 1 become unendowed insiders in period 2. We adopt the following notation: in period $t = 1, 2$, x_{et}, x_{ut}, and y_t denote period t consumption levels of endowed insiders, unendowed insiders, and outsiders, respectively. Note that, some agents "transition" from one category to another across periods (e.g., agents who were unendowed insiders in period 1 may become endowed insiders in period 2, etc.). Preferences are the same as in the main model. In each period, insiders can buy/sell housing and occupy it, whereas outsiders may only rent. Agents who own housing at the end of the first period (which they had consumed and/or rented out) retain this as their period 2 endowment. In each period, there is a sales price p_t and a rental price r_t. We now analyze our two benchmarks and then solve for equilibrium.

C.1 The Impact of Ownership Restriction

Note that, in both benchmark, it can only be harmful for unendowed and endowed insiders to have different consumption levels. Thus, here we write x_t $(= x_{et} = x_{ut})$. The "Second-Best" benchmark now solves $\max_{x_1,y_1,x_2,y_2} n_i u(x_1) + n_o \theta u(y_1) - n_o (1-\lambda) r_1 y_1 + (n_i + (1-\pi)n_o) u(x_2) + \pi n_o \theta u(y_2) - \pi n_o (1-\lambda) r_2 y_2$, subject to $\theta u'(y_t) = r_t$, $u'(x_t) = p_t$, $n_i x_1 + n_o y_1 = H$, and $(n_i + (1-\pi)n_o) x_2 + \pi n_o y_2 = H$. The solution is

$$u'(x_t^B) = \theta u'(y_t^B) - (1-\lambda)\theta \left(u'(y_t^B) + u''(y_t^B) y_t^B\right). \tag{C.1}$$

"Consumption-Optimal" solves $\max_{x_1,y_1,x_2,y_2} n_i u(x_1) + n_o \theta u(y_1) + (n_i + (1-\pi)n_o) u(x_2) + \pi n_o \theta u(y_2)$, which yields

$$u'(x_t^C) = \theta u'(y_t^C) \quad . \tag{C.2}$$

Recall, $MR\,(y_t) \equiv r_t + y_t \frac{dr_t}{dy_t}$ denotes the *Marginal Revenue* from rental in period t.

We now state Lemma 1†, which is analogous, in this model to Lemma 1 in the main text, and we follow this labeling convention throughout Appendix C. Propositions 8 and 9, which appear in Subsection 5.2, are proved at the end.

Lemma 1†. *When $MR\left(y_t^B\right) > 0$, the authority favors sales over rental, under the Second-Best regime, compared to the outcome under the Consumption-Optimal regime. When $MR\left(y_t^B\right) < 0$, the opposite is true, and, when $MR\left(y_t^B\right) = 0$, the allocations under the two regimes are identical. Formally,*

$$\operatorname{sign}\left\{u'(x_t^C) - u'(x_t^B)\right\} \;=\; \operatorname{sign}\left\{x_t^B - x_t^C\right\} \;=\; \operatorname{sign}\left\{y_t^C - y_t^B\right\} \;=\; \operatorname{sign}\left\{MR\left(y_t^B\right)\right\}.$$

Equilibrium Under Restricted Ownership. We solve backwards for equilibrium, as follows.

- Period 2's demand and supply conditions are identical to those of the main model. Namely, $\theta u'(y_2^*) = r_2^*$, $u'(x_{u2}^*) = u'(x_{e2}^*) = p_2^* = \lambda r_2^*$. The market clearing condition is $n_i x_{e2}^* + (1-\pi) n_o x_{u2}^* + \pi n_o y_2^* = H$.
- In period 1, outsiders' demand satisfies $\theta u'(y_1^*) = r_1^*$. For both types of insiders, the value of carrying any unit into the second period is $p_2^* = u'(x_{u2}^*) = u'(x_{e2}^*)$. Therefore, for unendowed insiders, we have $u'(x_{u1}^*) + p_2^* = p_1^*$, and, for endowed insiders, we have $u'(x_{e1}^*) + p_2^* = p_1^*$. On the supply side, for an endowed insider, renting out one unit in

period 1 brings λr_1^* while preserving value p_2^* for the next period. Hence, $p_1^* = \lambda r_1^* + p_2^*$. The market clearing condition is $n_e x_{e1}^* + n_u x_{u1}^* + n_o y_1^* = H$.

At equilibrium (when there is no sales taxes), $x_{et}^* = x_{ut}^* = x_t^*$, giving

$$u'(x_t^*) = \lambda\theta u'(y_t^*),\ t = 1, 2. \tag{C.3}$$

Proposition 1†. *Compared to benchmark C, which maximizes total utility from housing consumption, equilibrium allocates less housing to rental and more to ownership. Moreover, these outcomes satisfy*

$$y_t^* < y_t^C,\ x_t^C < x_t^*. \tag{C.4}$$

Proposition 2†. *Compared to benchmark B, which maximizes total surplus, equilibrium allocates less housing to rental and more to ownership. That is, $y_t^* < y_t^B$ and $x_t^B < x_t^*$, but a complete ranking cannot be guaranteed.*

C.2 The Effects of Taxation Under Ownership Restriction

C.2.1 Property Tax

As in the main model, a property tax has no effect on the various levels of housing consumption but extracts wealth from endowed insiders. This can be shown using the same logic found in the proof of Claim 1 in Appendix A.

C.2.2 Sales Tax

Suppose that there is a period-specific sales tax, τ_t. Then, at equilibrium,

$$\begin{aligned}
&u'(x_{e2}^*) = (1-\tau_2)p_2^*,\ u'(x_{u2}^*) = p_2^*,\ (1-\tau_2)p_2^* = \lambda r_2^*,\ \theta u'(y_2^*) = r_2^*,\ n_i x_{e2}^* + (1-\pi)n_o x_{u2}^* + \pi n_o y_2^* = H\\
&u'(x_{e1}^*) + (1-\tau_2)p_2^* = (1-\tau_1)p_1^*,\ u'(x_{u1}^*) + (1-\tau_2)p_2^* = p_1^*,\ \theta u'(y_1^*) = r_1^*,\ (1-\tau_1)p_1^* = \lambda r_1^* + (1-\tau_2)p_2^*,.\\
&n_e x_{e1}^* + n_u x_{u1}^* + n_o y_1^* = H
\end{aligned} \tag{C.5}$$

Total surplus is

$$\begin{aligned}
&\underbrace{n_e u\left(x_{e1}^*(\tau_t)\right) + n_u u(x_{u1}^*(\tau_t)) + n_o\theta u\left(y_1^*(\tau_t)\right) + n_i u(x_{e2}^*(\tau_t)) + (1-\pi)n_o u(x_{u2}^*(\tau_t)) + \pi n_o\theta u(y_2^*(\tau_t))}_{\text{utility from housing}}\\
&-\underbrace{\left(n_o(1-\lambda)\theta u'(y_1^*(\tau_t))y_1^*(\tau_t) + \pi n_o(1-\lambda)\theta u'(y_2^*(\tau_t))y_2^*(\tau_t)\right)}_{\text{management frictions}}.
\end{aligned} \tag{C.6}$$

Proposition 3†. *Starting from $\tau_t = 0$, a marginal increase in the period t sales tax rate*

(a) leads to an increase in management frictions if and only if $MR(y_t^*) = r_t^* + y_t^* \frac{dr_t}{dy_t}\big|_{y_t=y_t^*} > 0$,

(b) brings about an increase in total utility from housing consumption,

(c) gives rise to an increase in total surplus.

Proof. Our argument uses the following expressions, obtained by totally differentiating equation (C.5)

$$\frac{dy_2^*}{d\tau_1} = 0,$$

$$\frac{dy_2^*}{d\tau_2} = -\frac{(1-\pi)n_o u'(x_{u2}^*)u''(x_{e2}^*)}{n_o u''(x_{e2}^*)\left[(1-\pi)\lambda\theta u''(y_2^*) + \pi(1-\tau_2)u''(x_{u2}^*)\right] + n_i(1-\tau_2)\lambda\theta u''(x_{u2}^*)u''(y_2^*)} > 0,$$

$$\frac{dy_1^*}{d\tau_1} = -\frac{n_u u''(x_{e1}^*)\left(u'(x_{u1}^*) + \lambda\theta u'(y_2^*)\right)}{(1-\tau_1)u''(x_{u1}^*)\left[n_e\lambda\theta u''(y_1^*) + n_o u''(x_{e1}^*)\right] + n_u\lambda\theta u''(x_{e1}^*)u''(y_1^*)} > 0,$$

$$\frac{dy_1^*}{d\tau_2} = -\frac{n_u\lambda\theta u''(x_{e1}^*)u''(y_2^*)}{(1-\tau_1)u''(x_{u1}^*)\left[n_e\lambda\theta u''(y_1^*) + n_o u''(x_{e1}^*)\right] + n_u\lambda\theta u''(x_{e1}^*)u''(y_1^*)}\left(\tau_1\frac{dy_2^*}{d\tau_2}\right).$$

For part (a), the derivative of the second term of (C.6) with respect to τ_1, evaluated at $\tau_t = 0$, is

$$n_o(1-\lambda)\theta\left(u'(y_1^*) + u''(y_1^*)y_1^*\right)\frac{dy_1^*}{d\tau_1}.$$

The derivative of the second term of (C.6) with respect to τ_2, evaluated at $\tau_t = 0$, is

$$\pi n_o(1-\lambda)\theta\left(u'(y_2^*) + u''(y_2^*)y_2^*\right)\frac{dy_2^*}{d\tau_2}.$$

For part (b), the derivative of the first term of (C.6) with respect to τ_1, evaluated at $\tau_t = 0$, is

$$n_o(1-\lambda)\theta u'(y_1^*)\frac{dy_1^*}{d\tau_1} > 0.$$

The derivative of the first term of (C.6) with respect to τ_2, evaluated at $\tau_t = 0$, is

$$\pi n_o(1-\lambda)\theta u'(y_2^*)\frac{dy_2^*}{d\tau_2} > 0.$$

For part (c), the derivative of equation (C.6) with respect to τ_1, evaluated at $\tau_t = 0$, simplifies to

$$-n_o(1-\lambda)\theta u''(y_1^*)y_1^*\frac{dy_1^*}{d\tau_1} > 0.$$

The derivative of equation (C.6) with respect to τ_2, evaluated at $\tau_t = 0$, simplifies to

$$-\pi n_o(1-\lambda)\theta u''(y_2^*)y_2^*\frac{dy_2^*}{d\tau_2} > 0.$$

□

C.2.3 Rental Tax

Suppose that there is a period-specific rental tax, σ_t. Then, at equilibrium,

$$\begin{array}{l} u'(x_2^*) = p_2^*,\ p_2^* = (\lambda - \sigma_2)r_2^*,\ \theta u'(y_2^*) = r_2^*,\ \ (n_i + (1-\pi)n_o)\,x_2^* + \pi n_o y_2^* = H \\ u'(x_1^*) + p_2^* = p_1^*,\ \theta u'(y_1^*) = r_1^*,\ p_1^* = (\lambda - \sigma_1)r_1^* + p_2^*,\ n_i x_1^* + n_o y_1^* = H \end{array}. \tag{C.7}$$

Total surplus is

$$\begin{aligned} &\underbrace{n_i u(x_1^*(\sigma_t)) + n_o\theta u(y_1^*(\sigma_t)) + (n_i + (1-\pi)n_o)\,u(x_2^*(\sigma_t)) + \pi n_o\theta u(y_2^*(\sigma_t))}_{\text{utility from housing}} \\ &-\underbrace{\Big(n_o(1-\lambda)\theta u'(y_1^*(\sigma_t))y_1^*(\sigma_t) + \pi n_o(1-\lambda)\theta u'(y_2^*(\sigma_t))y_2^*(\sigma_t)\Big)}_{\text{management frictions}}. \end{aligned} \tag{C.8}$$

Proposition 6†. *Starting from $\sigma_t = 0$, a marginal increase in the rental tax rate*

(a) leads to a decrease in management frictions if and only if $MR(y_t^*) > 0$,

(b) brings about a decrease in total utility from housing consumption,

(c) gives rise to a descrease in total surplus.

Proof. Our argument uses the following expressions, obtained by totally differentiating equation (C.7).

$$\frac{dy_2^*}{d\sigma_1} = 0,$$

$$\frac{dy_2^*}{d\sigma_2} = \frac{(n_i + (1-\pi)n_o)\,\theta u'(y_2^*)}{(\lambda - \sigma_2)\,(n_i + (1-\pi)n_o)\,\theta u'(y_2^*) + \pi n_o u''(x_2^*)} < 0,$$

$$\frac{dy_1^*}{d\sigma_2} = 0,$$

$$\frac{dy_1^*}{d\sigma_1} = \frac{n_i\theta u'(y_1^*)}{n_i(\lambda - \sigma_1)\theta u''(y_1^*) + n_o u''(x_1^*)} < 0.$$

For part (a), the derivative of the second term of (C.8) with respect to σ_1, evaluated at $\sigma_t = 0$, is

$$n_o(1-\lambda)\theta\left(u'(y_1^*) + u''(y_1^*)y_1^*\right)\frac{dy_1^*}{d\sigma_1}.$$

The derivative of the second term of (C.8) with respect to σ_2, evaluated at $\sigma_t = 0$, is

$$\pi n_o(1-\lambda)\theta\left(u'(y_2^*) + u''(y_2^*)y_2^*\right)\frac{dy_2^*}{d\sigma_2}.$$

For part (b), the derivative of the first term of (C.8) with respect to σ_1, evaluated at $\sigma_t = 0$, is

$$n_o(1-\lambda)\theta u'(y_1^*)\frac{dy_1^*}{d\sigma_1} < 0.$$

The derivative of the first term of (C.8) with respect to σ_2, evaluated at $\sigma_t = 0$, is

$$\pi n_o(1-\lambda)\theta u'(y_2^*)\frac{dy_2^*}{d\sigma_2} < 0.$$

For part (c), the derivative of equation (C.8) with respect to σ_1, evaluated at $\sigma_t = 0$, simplifies to

$$-n_o(1-\lambda)\theta u''(y_1^*)y_1^*\frac{dy_1^*}{d\sigma_1} < 0.$$

The derivative of equation (C.8) with respect to σ_2, evaluated at $\sigma_t = 0$, simplifies to

$$-\pi n_o(1-\lambda)\theta u''(y_2^*)y_2^*\frac{dy_2^*}{d\sigma_2} < 0.$$

□

Proposition 7†. *Let $\varepsilon(y_t^*) \equiv -r_t^*/\left(y_t^*\frac{dr_t}{dy_t}\right)$ denote the elasticity of demand for rented housing at equilibrium under the restricted ownership policy, given σ_t. The Second-Best can be obtained by imposing a rental subsidy satisfying $\hat{\sigma}_t = -(1-\lambda)/\varepsilon(y_t^*) < 0$. In particular,*

$$\begin{cases} \hat{\sigma}_1 < \hat{\sigma}_2, \text{ if } \varepsilon'(y_t^*) < 0 \\ \hat{\sigma}_1 = \hat{\sigma}_2, \text{ if } \varepsilon'(y_t^*) = 0 \\ \hat{\sigma}_1 > \hat{\sigma}_2, \text{ if } \varepsilon'(y_t^*) > 0 \end{cases}. \tag{C.9}$$

Proof. Denote total surplus, as expressed in (C.8), by $W(\sigma_1, \sigma_2)$. The optimal rental tax,

$$\hat{\sigma}_t = (1-\lambda)\frac{\theta u''(y_t^*)y_t^*}{\theta u'(y_t^*)},\ t = 1, 2$$

solves

$$\frac{\partial W(\sigma_1,\sigma_2)}{\partial \sigma_1} = -n_o(\lambda-\sigma_1)\theta u'(y_1^*)\frac{dy_1^*}{d\sigma_1} + \left[n_o\theta u'(y_1^*) - n_o(1-\lambda)\theta\left(u'(y_1^*) + u''(y_1^*)y_1^*\right)\right]\frac{dy_1^*}{d\sigma_1} = 0$$
$$\frac{\partial W(\sigma_1,\sigma_2)}{\partial \sigma_2} = -\pi n_o(\lambda-\sigma_2)\theta u'(y_2^*)\frac{dy_2^*}{d\sigma_2} + \left[\pi n_o\theta u'(y_2^*) - \pi n_o(1-\lambda)\theta\left(u'(y_2^*) + u''(y_2^*)y_2^*\right)\right]\frac{dy_2^*}{d\sigma_2} = 0$$

Evaluated at $\hat{\sigma}_t$, $u'(x_t^*) = (\lambda - \hat{\sigma})\,\theta u'(y_t^*)$, which is equivalent to (C.1). For equation (C.9), we have shown in (C.10) below that $\frac{dy_2^*}{d\pi} > 0$, and $y_1^* = y_2^*$ if and only if $\pi = 1$, which implies that when $0 < \pi < 1$, $y_1^* > y_2^*$. At optimal rental tax, $\operatorname{sign}\left(\frac{d\hat{\sigma}_t}{d\pi}\right) = \operatorname{sign}\left(\varepsilon'(y_t^*)\right)$. □

Proof of Proposition 8. Totally differentiating the equilibrium conditions in the second period, i.e., $p_2^* = u'(x_2^*) = \lambda\theta u'(y_2^*)$ and $(n_i + (1-\pi)n_o)\,x_2^* + \pi n_o y_2^* = H$, with respect to π, gives

$$\begin{aligned}\frac{dy_2^*}{d\pi} &= \frac{n_o(x_2^* - y_2^*)u''(x_2^*)}{\pi n_o u''(x_2^*) + (n_i + (1-\pi)n_o)\,\lambda\theta u''(y_2^*)} > 0,\\ \frac{dx_2^*}{d\pi} &= \frac{n_o\lambda\theta(x_2^* - y_2^*)u''(y_2^*)}{\pi n_o u''(x_2^*) + (n_i + (1-\pi)n_o)\,\lambda\theta u''(y_2^*)} > 0,\\ \frac{dp_2^*}{d\pi} &= \lambda\theta u''(y_2^*)\frac{dy_2^*}{d\pi} < 0.\end{aligned} \tag{C.10}$$

Totally differentiating the equilibrium conditions in the first period, i.e., $u'(x_1^*) = p_1^* - p_2^* = \lambda r_1^* = \lambda\theta u'(y_1^*)$ and $n_i x_1^* + n_o y_1^* = H$, gives $\frac{dp_1^*}{d\pi} - \frac{dp_2^*}{d\pi} = 0$. For part (a), (C.10) implies that $\frac{d}{d\pi}\left(\frac{p_1^*}{2}\right) = \frac{1}{2}\frac{dp_2^*}{d\pi} < 0$, and $\frac{d}{d\pi}\left(\frac{p_1^*}{r_1^*}\right) = \lambda\frac{d}{d\pi}\left(1 + \frac{r_2^*}{r_1^*}\right) = \frac{\lambda}{r_1^*}\theta u''(y_2^*)\frac{dy_2^*}{d\pi} < 0$; For part (b), combining (C.10) and part (a), $\frac{d}{d\pi}\left(p_2^* - \frac{p_1^*}{2}\right) < 0$; For part (c), the two-period resource constraints imply $n_i(x_1^* - x_2^*) = -n_o y_1^* + (1-\pi)n_o x_2^* + \pi n_o y_2^*$, the derivative of which, with respect to π, gives

$$n_i\frac{d}{d\pi}\left(x_1^* - x_2^*\right) = -\frac{n_i n_o\lambda\theta\left(x_2^* - y_2^*\right)u''(y_2^*)}{\pi n_o u''(x_2^*) + (n_i + (1-\pi)n_o)\,\lambda\theta u''(y_2^*)} < 0.$$

□

Proof of Proposition 9. For part (a), note that under no ownership restriction (first-best), $2u'\left(x^A\right) = p^A$ and $x^A = H/(n_i + n_o)$. With any level of $\pi \in [0,1]$, the equilibrium conditions imply that $x^A < x_2^* < x_1^*$. Starting from no restriction, the derivative of an unendowed insider's indirect utility, i.e., $2u(x^A) - 2u'(x^A)x^A$ with respect to x^A, gives $-2u''(x^A)x^A > 0$. The derivative of an endowed insider's indirect utility, i.e., $2u(x^A) - 2u'(x^A)(h_e - x^A)$, with respect to x^A, gives $2u''(x^A)(h_e - x^A) < 0$. Combining $x^A < x_2^* < x_1^*$, restricted ownership brings about an increase in the consumption level of insiders, which makes unendowed insiders better off and endowed insiders worse off.

The derivative of the indirect utility of an endowed insider, i.e., $u(x_1^*) + u(x_2^*) + p_1^*\left(h_e - x_1^*\right) +$

$p_2^*\left(x_1^* - x_2^*\right)$, with respect to π, gives $\left(h_e - x_2^*\right)\frac{dp_2^*}{d\pi} < 0$; the derivative of the indirect utility of an unendowed insider, i.e., $u(x_1^*) + u(x_2^*) - p_1 x_1^* + p_2^*\left(x_1^* - x_2^*\right)$, with respect to π, gives $-x_2^*\frac{dp_2^*}{d\pi} > 0$.

For part (b), the derivative of the indirect utility of an outsider in period 2, i.e., $\theta u'(y_2^*) - r_2^* y_2^*$, with respect to π, gives $-\theta u''(y_2^*) y_2^* \frac{dy_2^*}{d\pi} > 0$; the derivative of the indirect utility of an unendowed insider in period 2, with respect to π, gives $-x_2^*\frac{dp_2^*}{d\pi} > 0$. $\square$