Vertical Agreements and User Access[*]

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Abstract

Platforms acting as sales channels for producers often charge users for access, via a subscription fee or a markup on hardware. We compare two common forms of vertical pricing agreement that platforms use with sellers: per-unit and proportional fees. In particular, we analyze the critical role that user access plays on prices, profits, and welfare under both forms of agreement. We characterize this role and show how it potentially overturns standard results saying that proportional fees lead to lower prices and higher profits.

Keywords: Platforms, User Access, Per-Unit vs. Proportional Fees, Wholesale vs. Agency Agreements, Unit vs. *Ad Valorem* Taxation, Antitrust

JEL Codes: D21, D40, L23, L4, L42, L51, L82, L86

A number of recent articles compare variants of two forms of vertical pricing agreement, sometimes labelled "wholesale" and "agency." Primary events motivating these studies include controversies surrounding Apple and Amazon's contracts with media publishers, payment intermediaries' transaction fees and the structure of online travel agents' commissions.[1] A key distinguishing factor between these two forms of agreement is that, under wholesale, the intermediary or "platform" charges a constant *per-unit* markup for each item that producers sell, whereas, under agency, the platform receives a *proportional* share of the revenue from each sale.

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[1]On the first topic, see Abhishek, Jerath and Zhang (2016); Gaudin and White (2014*a*); Johnson (2017); Foros, Kind and Shaffer (2017), on the second, see Shy and Wang (2011); Wang and Wright (2017, 2018), and on the third, see Condorelli, Galeotti and Skreta (2018).

Two central results are that, holding other factors constant, agency agreements typically promote lower prices and are more profitable for platforms. The driving force behind these results is that, although both forms of agreement lead to a double-markup distortion in the pricing of goods, this is less severe under agency. The reason for the smaller double-markup distortion under agency is its proportional, rather than per-unit, transaction fees.

A crucial point, however, is that these results do not take into account the fact that platforms often charge users for access. For instance, consider the following two contrasting quotations. In 2003, then Apple CEO Steve Jobs had this exchange about the iTunes music store:

> *'The dirty little secret of all this is there's no way you can make money on these stores'...Why even bother? 'Because we're selling iPods.'*[2]

However, in 2012, Amazon's CEO, Jeff Bezos, said this about the firm's Kindle ebook platform:

> *We sell our hardware at cost.... We're not trying to make money on the hardware. We want to make money when people use our devices, not when people buy our devices.*[3]

Platforms can differ dramatically from one situation to another in the extent to which they earn profits from selling "user access" (e.g., hardware, subscription fees, etc.) versus "usage goods" (e.g., media, hotel rooms, event tickets, etc.). This paper compares agreements with per-unit and proportional transaction fees, taking into account the crucial point raised by the quotes above. Our main message is that user access critically affects this comparison in theoretically interesting ways. Moreover, when a platform has a high ability to sell access to users, this can reverse the aforementioned results regarding prices and platform profits.

Our model compares two types of vertical agreement between a platform and a producer. Under a per-unit "U-agreement," the markup taken by the platform is denoted by r, while p denotes the final price of the usage good, and c the producer's marginal cost. Under a proportional "A-agreement," the share of revenue retained by the platform is denoted by a, and, given the final usage good price p, the platform thus earns $a \cdot p$, whereas the producer earns $(1-a)\,p - c$. A key feature of the model is that the platform may charge users for access as a precondition to their purchasing the usage good, which the producer sells.

[2]Taylor (2003).

[3]Quoted from the Jeff Bezos interview with BBC News; see Cellan-Jones (2012).

In the previously studied setting with no user access charge, U-agreements typically give rise to a higher price for the usage good than A-agreements. The only time this fails to be true is when demand is "super-convex," according to the classification of Mràzovà and Neary (2017) – that is, when demand grows *more* elastic as quantity sold increases. Moreover, without a user access charge, the platform always earns greater profits under an A-agreement.

We establish two main sets of results. First, we derive a simple formula governing the ranking of prices for the usage good under the two forms of agreement. This generalizes the aforementioned condition, originally stated by Bishop (1968) in the context of commodity taxation, that hinged on Marshall's (1890) second law of demand. Under the generalized formula, as the platform charges more for user access, the possibility grows that a U-agreement gives rise to a lower price than an A-agreement. Now, when demand elasticity decreases with quantity, the A-agreement price may exceed the U-agreement one, so long as the rate at which elasticity decreases obeys a limit, which relaxes as the user access revenue stream becomes more important.

Second, we uncover a key link between the platform's optimal markups and its profits under the two regimes. Regarding the former, it is straightforward to see that, as the platform can earn more by charging users for access, it has, broadly speaking, less incentive to tax the usage good and, potentially, the incentive to subsidize it instead.[4] We show that, as the importance of user access grows, this incentive to subsidize builds up more quickly under a U-agreement. This means that, in a given demand environment (i.e., holding fixed user preferences for both usage and access), the platform may do one of three things: (1) it may charge positive transaction fees under both regimes, (2) it may charge a negative fee under a U-agreement but a positive fee under an A-agreement, or (3) it may charge negative fees under both regimes. No circumstances exist in which it would wish to tax under a U-agreement but subsidize under an A-agreement.

This result connects to the platform's profits in the following way. When user access plays a sufficiently small role, the platform taxes under both regimes and an A-agreement yields greater profits (as in existing models). In contrast, when the platform wishes to tax under an A-agreement but subsidize under a U-agreement, the latter may be more profitable. When the

[4]This is a standard theme in the literature on two-part tariffs (Oi, 1971; Schmalensee, 1981; Varian, 1989).

platform wishes to subsidize under both regimes, a U-agreement is always more profitable.

We carry out the analysis over two main sections. In the first (Section II), we model the role of user access in a simplified way, using an exogenous parameter, β. By taking this shortcut, we can devote full attention to identifying the set of key statistics related to the market for the usage good. Then, in Section III, we develop a more detailed version of the model in which the platform optimally coordinates its actions across the market for the good and the market for user access. Under this approach, β becomes endogenous, the statistics identified in the prior section retain their importance, and so do the main substantive economic findings.

A nice feature of Section III's endogenous approach is that it allows for a more comprehensive examination of welfare. This enables us to address questions regarding the alignment or misalignment of preferences among the platform, the producer, and users for one form of agreement or the other. Strikingly, we show that, when the user access market carries enough weight, from the platform's perspective, U-agreements can Pareto-dominate A-agreements. This result stands in particular contrast to most findings in the literature comparing these two forms of charging methods, going back to the celebrated result of Suits and Musgrave (1953) on the welfare superiority of *ad valorem* over per-unit commodity taxation.

At a high level, the paper clarifies our understanding of platforms' incentives and may help guide policy decisions in this area. As the above quotations, from Bezos and Jobs, suggest, Apple, *circa* 2003, had greater market power in selling iPods than Amazon, *circa* 2012, had in selling Kindles. In view of our model, the observed pattern makes sense: on the one hand, Apple's arrangements, at the time, with record companies were approximated by U-agreements, and it charged very small (sometimes negative) markups on music downloads, while its so-called "dirty little secret" was its considerable profits from selling iPods. On the other hand, Amazon's arrangements with book publishers were similar to A-agreements, and it touted usage goods, not devices, as the more significant source of its profits.

Regarding policy, with some simplification, our model can be summarized as saying that there are three "zones" governing the alignment between a platform's preference between the two forms of agreement and that of consumers. First, when user access is of "low" importance to the platform, it prefers an A-agreement, and so do consumers. Second, when user access is of “medium” importance, the platform prefers an A-agreement, but consumers prefer a U-agreement. Finally, when user access is of “high” importance, both the platform and consumers prefer a U-agreement.[5]

Taking this configuration into account, consider a situation in which an authority is considering restricting a platform from using A-agreements. In the 2013 antitrust case that the U.S. Department of Justice won against Apple regarding ebook pricing, for example, the judge imposed a such a restriction. The aforementioned medium zone in our model constitutes a novel rationale for favoring U-agreements. Indeed, De los Santos and Wildenbeest (2017) provide evidence that, following the switch from A- to U-agreements, ebook prices fell. However, the model also urges caution, because, if such an action were taken in the low zone, it would leave both the platform and consumers worse off. Our framework provides concrete theoretical guidance to distinguish between these two situations. U-agreements, in contrast, have the property that, whenever the platform prefers them, so must consumers, and so the model suggests no rationale for restricting them.

I Related Literature

Of course, the comparison between per-unit and proportional vertical pricing agreements depends on factors beyond those that we can incorporate into our model, some of which earlier literature highlights. The current paper is most closely related to a set of articles that draw motivation from the ebook industry and the associated antitrust case. Gilbert (2015) provides an early survey on this topic, discussing the relative properties of both types of pricing agreements, which, for instance, Foros, Kind and Shaffer (2017), Gaudin and White (2014*a*), and Johnson (2017) study theoretically and De los Santos and Wildenbeest (2017) examine empirically.[6]

More specifically, Foros, Kind and Shaffer (2017) and Johnson (2017) investigate the impact of of “Most Favored Nation” clauses (MFN) on either agency and wholesale agreements,[7] whereas Gaudin and White (2014*a*) consider the effects of increased competition in the tablet market on

[5]For the purposes of any policy considerations, it should be borne in mind that our model abstracts away from other, potentially important dimensions of platforms’ behavior and incentives, some of which are covered by the literature cited in Section I.

[6]See also, Reimers and Waldfogel (2017).

[7]See also, Boik and Corts (2016).

ebook pricing. Other models specifically tailored toward the ebook industry are Adner, Chen and Zhu (2020), studying the compatibility decisions of Apple and Amazon, and Johnson (2020), focusing on consumer switching costs. In a related analysis, Abhishek, Jerath and Zhang (2016) consider externalities for a producer that sells both through traditional and online channels.[8]

Other related works focus particularly on payment intermediaries (Shy and Wang, 2011) and other types of platforms. Importantly, Condorelli, Galeotti and Skreta (2018), Hagiu and Wright (2015, 2019), and Wang and Wright (2017, 2018) emphasize a set of issues related to asymmetric information and price discrimination, which are absent from our model but which can also play a crucial role in the comparison between forms of agreement. Gans (2012), and Llobet and Padilla (2016) address related issues pertaining to the timing of pricing of various goods and services in digital markets, and patent licensing and its impact on innovation, respectively, while Gomes and Tirole (2018) study situations where consumers are not fully informed about platforms' pricing agreements. Finally, Chen and Rey (2019) study the role played by cross-subsidization when multi-product firms with a different competitive advantage face both one-stop shoppers and multi-stop shoppers. However, none of these articles analyze the impact of the platform externalities induced by user access under a flexible form of demand.

In comparing per-unit and proportional agreements, our work also builds on the classic taxation literature contrasting unit and *ad valorem* commodity tax regimes (Suits and Musgrave, 1953; Skeath and Trandel, 1994; Anderson, de Palma and Kreider, 2001*a*,*b*). Within the taxation literature, Bishop (1968) provides important insight in the case where an authority maximizes tax revenues, which Gaudin and White (2014*b*) and Gu, Yao and Zhou (2019) build on. Miravete, Seim and Thurk (2018) empirically examine whether commodity taxes do indeed maximize government revenues.

Our model pays close attention to the effect of demand curvature on comparative statics, and, in doing so, it relies particularly on Bulow and Pfleiderer (1983), Weyl and Fabinger (2013) and Mràzovà and Neary (2017).[9] We point out specific connections when appropriate. We also help link such work to classic two-part tariff analysis (Oi, 1971; Schmalensee, 1981; Varian, 1989).

[8]Their analysis, however, is restricted to the case of a linear demand form, and intermixes both changes in contractual agreements and changes in the order of moves for the players.

[9]Other recent papers that rely on such demand primitives to interpret their results include Aguirre, Cowan and Vickers (2010), and Chen and Schwartz (2015), which compare uniform versus differential pricing regimes.

II A Simple Model

II.A Setup

Consider a model with three types of agents: a platform, a producer and a mass of consumers. The producer makes a good that consumers can enjoy in variable quantities (hereafter, "the good"). In order to reach consumers, the producer must sell via the platform.

The two forms of contracting agreement between the platform and the producer that we are interested in studying are *U-agreements* and *A-agreements*, indexed by subscript $i = U, A$. Under the former, the platform sets a constant, per-unit markup, whereas, under the latter, it sets a fraction of revenue to collect. Moreover, we assume that the platform also has the ability to control "user access." That is, it can require consumers to pay a fee to "join the platform" as a precondition to accessing the good. This fee could perhaps be linked to the sale of a physical hardware device, or it could be tied to a virtual subscription.

We denote the per-unit price of the good by p. In this section, we represent consumers simply by their aggregate demand function for the good, $q(p)$. The function $q(\cdot) : \mathbb{R}_{++} \to \mathbb{R}_{+}$ is assumed, throughout the part of its domain for which it takes non-zero values, to be strictly decreasing and three times continuously differentiable. Wherever $q(\cdot)$ is invertible, let $p(q)$ denote inverse demand, let $mr(q) \equiv p(q) + q\,p'(q)$ denote marginal revenue, and let $s(q) \equiv \int_0^q p(z)\,dz - q\,p(q)$ denote consumer surplus from purchasing the good. In Appendix A.1, we provide second-order conditions. Moreover, throughout the paper, we use the following statistics to characterize demand.

Definition 1 (Key demand statistics)**.** *Evaluated at a given quantity, q, define the following local statistics pertaining to the demand for the good:*

(a) $\varepsilon(q) \equiv -\frac{p(q)}{q\,p'(q)}$ *denotes the* price-elasticity*,*

(b) $\sigma(q) \equiv -\frac{q\,p''(q)}{p'(q)}$ *denotes the* curvature*, and*

(c) $\delta(q) \equiv \frac{p'(q)}{mr'(q)} = \frac{1}{2-\sigma(q)}$ *denotes the cost* pass-through rate *faced by a hypothetical, integrated monopolist.*

Both (b) and (c) appear frequently in the literature on pricing under monopoly and imperfect competition and have a one-to-one mapping to one another. Whereas the former (emphasized, for instance, in Aguirre, Cowan and Vickers (2010) and Mràzovà and Neary (2017)) deals directly with the level of a demand curve's concavity/convexity, the latter, discussed extensively by Bulow and Pfleiderer (1983) and Weyl and Fabinger (2013), has intuitive value, as it measures the optimal rate at which a monopolist, currently maximizing its profits, would increase its price, in response to a small increase in its marginal cost. As is apparent from the right-most expression in (c), (inverse) demand that is more convex (i.e., with a greater $\sigma(\cdot)$) features a higher pass-through rate. Note, also, that demand that is locally strictly log-concave (i.e., with $\sigma(q) < 1$) has a pass-through rate strictly below one, whereas, otherwise, it has a pass-through rate greater than or equal to one.[10]

Regarding firms' technology, the crucial parameter to account for is the producer's marginal cost, which we assume to be constant, denoted by $c \in (0, \bar{c})$, where $\bar{c}$ is small enough to ensure positive demand in equilibrium. In the main text, we assume the platform's marginal cost of distributing the good to be zero, and we relax this in Appendix C.1.

Under a U-agreement, r denotes the (potentially negative) per-unit markup set by the platform, and p denotes the final price for the good set by the producer. The producer earns profits $(p - r - c)\,q$. The platform earns profits both from selling the good and from selling user access, and these, in total, are given by $r \cdot q + \beta \cdot s\,(q)$. Under an A-agreement, the producer sets p, and the platform retains fraction a (potentially negative) of the revenue from sales of the good. The producer's profits are thus $[(1 - a)\,p - c]\,q$, and the platform's profits are $a \cdot p \cdot q + \beta \cdot s\,(q)$.

$\beta \geq 0$ measures the level of importance played by user access in the platform's profits. In this section, because of our focus on identifying key aspects of *the market for the good*, we take this to be an exogenous parameter, and then we endogenize it in Section III. For the purposes of this section, β can be motivated as follows.

On the one hand, when the platform cannot charge users for access, i.e., $\beta = 0$, the environment is one of linear pricing for the good. On the other hand, a second focal case, in which $\beta = 1$, is that of a "textbook" two-part tariff in which consumers all have the same demand

[10] The pass-through rate is strictly positive, because the second-order conditions, detailed in Appendix A.1, imply $2 > \sigma(q)$, $\forall q$.

curve for the good, and the platform has monopoly power over user access. In this case, by charging for access, the platform can extract all of the surplus that users derive from consuming the good. β serves a simple way to approximate situations that might arise more generally (for example, where Amazon sells Kindles and Prime subscriptions, or Apple sells iPods, iPhones, iPads, etc.). Thus, for the purposes of this section, it suffices to think of β as a rough measure of the platform's market power over user access; when it takes on high values, we say that user access "plays an important role." Typically, β would likely be between zero and one, as arises endogenously in Section III, though for expositional simplicity in this section, we do not impose any upper bound.

In Section III, we fully specify the market for user access and show how a given state in that market maps to a particular value of β. As that section shows, a restrictive aspect of this section's approach is that β is held constant *across agreement types*. As such, here we consider just the platform's choice of r under a U-agreement and a under and A-agreement while ignoring more structural adjustments it might make. For example, the current setting could represent a situation where the platform has already committed to selling a certain number of hardware devices, and where the price of this device will adjust to clear the market, as a function of the price of the good. In Section III, the key additional step is to allow the platform to choose its optimal supply in the user access market, under each form of agreement. As will become clear, the demand conditions identified in this section retain their relevance in that environment.[11]

Using subgame perfect Nash equilibrium as our solution concept, we assume the following timing under the two forms of agreement:

- **U-agreement**
    1. The platform sets its per-unit fee, r,
    2. The producer sets the final price for the good, p;
- **A-agreement**
    1. The platform sets its revenue share, a,
    2. The producer sets the final price for the good, p.

The assumption that the platform moves first reflects the idea that, in the contexts we have in

[11] Also see Appendix C.2, which replaces β with a more general function.

mind, the platform is the most powerful player,[12] dictating its own pricing strategy and leaving others to respond. Our timing holds fixed this bargaining power across regimes and compares the incentives created by constant-markup versus proportional revenue-sharing arrangements.[13]

II.B User Access, Demand Curvature, and Equilibrium Pricing

This subsection considers equilibrium pricing under the two forms of agreement. It shows that (a) a more significant role for user access as a source of revenue for the platform and (b) more convex demand can lead prices to be lower under U-agreements than A-agreements. It then shows that these forces may lead the platform to subsidize rather than tax the good, and that this occurs more often under U-agreements.

A simple way to solve the game involves expressing each firm's action as a choice of the quantity of the good to be sold. First consider a U-agreement. In the final stage, the producer's profits are equal to $[p(q) - (c + r)]\, q$, giving rise to first-order condition

$$mr(q) - c = r. \tag{1}$$

We then use equation (1) to substitute for r in the platform's objective function, which becomes

$$\max_q \,(mr(q) - c)\, q + \beta\, s(q). \tag{2}$$

Now consider an A-agreement. In the final stage, the producer maximizes $[(1 - a)\, p(q) - c]\, q$, giving rise to first-order condition

$$\frac{mr(q) - c}{mr(q)} = a. \tag{3}$$

[12] See, for instance, chapter 8 of Stone (2013), describing Amazon's negotiations with publishers leading up to the launch of the Kindle platform.

[13] Some works comparing "wholesale" and "agency" agreements simultaneously vary the form of the transaction fee (i.e., per-unit versus proportional) and the order of moves (See, e.g. Gaudin and White (2014*a*); Johnson (2017)). That is, they assume that, under wholesale agreements, the upstream "supplier" moves first and "sells" the good to the downstream "retailer," whereas, under agency, the retailer sets a proportional fee and leaves final pricing control to the supplier. In this paper, our goal is to isolate the role of different forms of transaction fee under general demand.

Using equation (3) to substitute for a in the platform's profits gives objective function

$$\max_{q} \frac{mr(q)-c}{mr(q)} p(q)\, q + \beta\, s(q)\,. \tag{4}$$

Lemma 1 regards the platform's first-order conditions under the two regimes. All proofs are relegated to Appendix B.

Lemma 1. *For $i = U, A$, denote by $\pi_i(q)$ the profits earned by the producer as a function of the platform's strategic choice. Under both forms of agreement, the equilibrium quantity sold, q_i, satisfies*

$$mr(q_i) - c = -\beta\, s'(q_i) + \pi_i'(q_i)\,, \tag{5}$$

where $\pi_U'(q) = -mr'(q)\, q$ and $\pi_A'(q) = -mr'(q)\, q \frac{p(q)/mr(q)}{mr(q)/c}$.

This lemma identifies two opposing forces whose combined impact determines whether the platform chooses a quantity smaller or greater than what an integrated monopolist (not selling user access) would pick, as specified in Definition 2 below.

Definition 2 (Integrated monopoly quantity)**.** *Let $q^M \equiv \arg\max_q \{(p(q)-c)\, q\}$, i.e., $mr(q^M) - c = 0$.*

On the one hand, $-\beta\, s' < 0$ represents the incentive to expand output in order to increase consumers' surplus from the good which is to be extracted via the sale of user access. On the other hand, $\pi_i' > 0$ reflects the fact that, under either of these vertical arrangements, there is an incentive for double marginalization. For a given output level, q, the magnitude of the former incentive does not depend on the agreement type, whereas the magnitude of the latter does: it is stronger under an A-agreement than it is under a U-agreement if and only if $p(q)/mr(q) > mr(q)/c$.

II.B.1 Price Comparison

Making use of Assumption 1, Lemma 2 establishes a unique threshold that determines whether $p(q)/mr(q) > mr(q)/c$ holds.

Assumption 1. $\frac{d}{dq}\left\{p(q)/mr(q)^2\right\} > 0 \;\Leftrightarrow\; \delta(q) < 2\, p(q)/mr(q),$[14] $\;\forall q \leq q^M$.

[14]To see this equivalence, note that $\frac{d}{dq}\{p(q)/mr(q)^2\} = \frac{-mr'(q)}{mr(q)^2}\left(\frac{2\,p(q)}{mr(q)} - \delta(q)\right)$. Also equivalent is $2\,\sigma(q) < 3 + 1/\varepsilon(q)$.

This assumption, which constrains the convexity of demand, is very weak, as it is compatible with all commonly used functional forms of which we are aware. In particular, constant elasticity demand (discussed below in Example 1) globally satisfies $\delta(q) = p(q)/mr(q)$. Therefore, even demand that is strictly more convex than this form is accommodated. We suppose that Assumption 1 holds throughout the remainder of the paper.

Lemma 2. *In the interval $\left(0, q^M\right)$, there exists a unique q^0, such that $p\left(q^0\right)/mr\left(q^0\right) = mr\left(q^0\right)/c$. Moreover, $q < q^0 \Leftrightarrow p(q)/mr(q) < mr(q)/c$, for all q such that $mr(q) > 0$.*

Lemma 2 allows for straightforward comparison of the double marginalization incentives under the two forms of agreement. At q^0, they are of equal strength, whereas, for all $q < q^0$ this incentive is stronger under a U-agreement, and vice versa. Now consider the role of user access, as captured in this section by β.

Definition 3. *Let $\beta^0 \equiv \frac{-q^0 \varepsilon'\left(q^0\right)}{\varepsilon\left(q^0\right)}$ denote the* quantity-neutral *level of β.*

Remark. *Later, we make use of these equivalences: $\frac{-q\varepsilon'(q)}{\varepsilon(q)} = \frac{1}{\delta(q)} - \frac{mr(q)}{p(q)} = 1 + \frac{1}{\varepsilon(q)} - \sigma(q)$.*[15]

Proposition 1 now compares equilibrium prices under the two regimes. Recall that q_i denotes the equilibrium quantity under an agreement of form $i = U, A$.

Proposition 1. *If $\beta < \beta^0$, then $q_U < q_A < q^0$; if $\beta = \beta^0$, then $q_U = q_A = q^0$; and if $\beta > \beta^0$, then $q_U > q_A > q^0$. Therefore, the good's price is lower under a U-agreement than under an A-agreement whenever β is greater than the quantity-neutral level.*

To interpret Proposition 1, it is instructive to first consider the following special case, which is roughly equivalent to a result stated in Bishop's (1968) work on commodity taxation.[16]

Corollary 1 (Bishop (1968))**.** *Assume the platform earns no profits from selling user access; that is, $\beta = 0$. The price of the good is greater under a U-agreement than under an A-agreement if and only if $\varepsilon'\left(q^0\right) < 0$.*

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[15]To derive these, note that $\varepsilon'(q) = \frac{-\varepsilon(q)}{q}\left(1 + \frac{1}{\varepsilon(q)} - \sigma(q)\right)$ and $\frac{1}{\delta(q)} = 2 - \sigma(q)$.

[16]Bishop's article focuses on comparing *ad valorem* and unit taxation, but it focuses only briefly (see p. 207) on the scenario in which the tax authority seeks to maximize revenue in a manner analogous to the platform in our model, and it does not use our technique involving q^0. In Gaudin and White (2014*b*), we analyze such a taxation model under imperfect competition.

This corollary with $\beta = 0$ illustrates a clear threshold, regarding the curvature of demand: so long as demand is not too convex, the forces driving double marginalization are greater under a U-agreement, leading to a higher price. According to the classification of Mràzovà and Neary (2017), which we adopt below, demand that globally satisfies $\varepsilon'(q) \leq 0$ is "sub-convex,"[17] whereas demand that globally satisfies $\varepsilon'(q) \geq 0$ is "super-convex."

Now, consider the more general case of Proposition 1, where $\beta \geq 0$. The comparison between prices under the two forms of agreement continues to depend on demand curvature. However, as β increases, demand that is less convex may still yield a higher price under an A-agreement. To see this, note that the condition $\beta > \beta^0$, discussed in Proposition 1, can be rewritten as

$$\beta + \sigma(q^0) > 1 + \frac{1}{\varepsilon(q^0)}. \tag{6}$$

The following two examples further illustrate the roles of user access and demand curvature in determining the price comparison under the two forms of agreement. Both involve commonly used demand forms, which Bulow and Pfleiderer (1983) point out have parameters that capture their curvatures.

Example 1 (Constant elasticity demand)**.** *Let $p(q) = \alpha\, q^{-1/\varepsilon}$, where $\alpha > 0$ and the constant elasticity parameter, $\varepsilon > 1$. Since $\varepsilon'(q) = 0$, from Proposition 1, $\beta^0 = 0$, i.e., the price under a U-agreement is strictly lower than under an A-agreement if and only if $\beta > 0$. Under this specification, curvature is $\sigma = 1 + 1/\varepsilon > 1$, and the pass-through rate is $\delta = \varepsilon/(\varepsilon - 1) > 1$.*

Example 2 (Log-concave constant pass-through demand)**.** *Let $p(q) = \alpha - \theta\, q^{\mu}$, where $\alpha > c$, $\theta > 0$ and $\mu > 0$. Here, $\sigma = 1 - \mu < 1$, and $\delta = 1/(1 + \mu) < 1$. Using equation* (6)*, it follows that the price under a U-agreement is strictly lower than under an A-agreement if and only if $\beta - \mu > 1/\varepsilon(q^0)$.*

Figure 1, below, helps to visualize the behavior of β^0 in these examples.

II.B.2 Tax Versus Subsidy

In the absence of revenue from user access, the platform's optimal choice, under either regime, is clearly to tax the good, rather than to subsidize it. On the other hand, when $\beta > 0$, this ceases to

[17] This property is also known as Marshall's (1890) second law of demand.

be obvious and, indeed, may no longer be the case. We now derive the conditions determining this issue, starting with Definition 4.

Definition 4. *(a) Let* $\beta_U^M \equiv \frac{1}{\delta(q^M)}$ *denote the* no-markup *level of* β *under a U-agreement, and*

(b) let $\beta_A^M \equiv \frac{p(q^M)/c}{\delta(q^M)}$ *denote the* no-markup *level of* β *under an A-agreement.*

Note that, since $\delta(q^M) > 0$ and $p(q^M) > c > 0$, it holds that $\beta_A^M > \beta_U^M > 0$. Further note the following Lemma, on which we rely in Section III.

Lemma 3. *If demand is globally, strictly*

(a) (i) log-concave ($\sigma(q) \leq 1, \forall q$), then $\beta_U^M > 1$, (ii) log-convex ($\sigma(q) \geq 1, \forall q$), then $\beta_U^M < 1$;

(b) (i) sub-convex ($\varepsilon'(q) \leq 0$), then $\beta_A^M > 1$, (ii) super-convex ($\varepsilon'(q) \geq 0$), then $\beta_A^M < 1$.

We now state Proposition 2 regarding the sign of the platform's markups.

Proposition 2. *Under an agreement of type $i = U, A$, when $\beta = \beta_i^M$, it is optimal for the platform to neither tax nor subsidize the good, inducing the producer to choose q^M. Furthermore,*

(a) if $\beta < \beta_U^M$, then the platform taxes under both forms of agreement ($r, a > 0$);

(b) if $\beta \in \left(\beta_U^M, \beta_A^M\right)$, then the platform subsidizes under a U-agreement and taxes under an A-agreement ($r < 0 < a$);

(c) if $\beta > \beta_A^M$, then the platform subsidizes under both forms of agreement ($r, a < 0$).

No primitives exist that lead the platform to tax in a U-agreement but subsidize in an A-agreement.

To understand Proposition 2, consider the platform's respective profit maximization problems under the two forms of agreement, rewritten in the following way. Let $P_U(r)$ and $P_A(a)$ denote the producer's best-response functions, that is, the prices it sets, respectively, given r or a, under the respective agreements. For a U-agreement, write the platform's problem as $\max_r r \cdot q(P_U(r)) + \beta \int_{P_U(r)}^{\infty} q(z)dz$, and consider the question of whether, starting from $r = 0$, it has an incentive to deviate in the direction of a tax or a subsidy. The derivative of this expression is

$$q^M - \beta\delta(q^M)q^M. \tag{7}$$

The first term in (7) captures the change in revenue that the platform obtains via sales of the good. The second term reflects the change in revenue that the platform obtains from users, and it contains three factors: β is the fraction of consumer surplus that the platform extracts, $\delta(q^M) = P'_U(0)$ is the rate at which the producer increases its price in response to the introduction of the tax,[18] and q^M is the reduction in consumer surplus per unit of price increase.

Thus, when $\beta = \beta_U^M = 1/\delta(q_M)$, the expression in (7) equals zero, and the platform has no incentive to deviate from $r = 0$. Moreover, for a given value of β, when the pass-through rate is sufficiently low, the platform prefers to tax, because the producer dampens the (negative) effect on consumers. Meanwhile, when the pass-through rate is sufficiently high, the platform prefers to subsidize, because the producer amplifies the positive effect on consumers.

Under an A-agreement, the same basic pattern applies, namely that demand with low pass-through tends to favor taxation, whereas demand with high pass-through tends to favor subsidizing the good. However, under an A-agreement, the threshold level of pass-through at which it is optimal to switch from tax to subsidy is higher than under a U-agreement. To see why, write the platform's problem as $\max_a a \cdot P_A(a) \cdot q(P_A(a)) + \beta \int_{P_A(a)}^{\infty} q(z)dz$. Analogously to the exercise above, start from $a = 0$ and consider what effect a small deviation has on the sign of the platform's profits. The derivative of this expression is

$$p(q^M)q^M - c\delta(q^M)q^M\beta. \tag{8}$$

As was true under a U-agreement, the first term in (8) captures the change in revenue that the platform obtains from sales of the good, while the second term captures the change in revenue it obtains from consumers. However, compared to the analogous derivative under a U-agreement, each of these two terms is multiplied by an additional factor. In the first term, the factor $p(q^M)$ reflects the fact that the platform's earnings via sales of the good are proportional to the producer's revenue, rather than the number of units sold. Meanwhile, in the second term, c reflects the fact that $P'_A(0) = c\delta(q^M)$.[19] That is, when a proportional tax is introduced,

[18] To see this, note that $\delta(q) = \frac{dp/dq}{dmr/dq} = \frac{dp/dq}{d(c+r)/dq} = \frac{dP_U}{dr} = P'_U(r)$.

[19] To derive this, rewrite the producer's first-order condition in equation (3) as $mr(q) - \frac{c}{1-a} = 0$ and totally differentiate with respect to a.

the producer perceives an increase in its marginal cost that is proportional to that cost's initial value, c.

As $p(q^M) > c$, a small, positive proportional fee is relatively more efficient than a small, positive unit fee, in that it brings in revenue from sales of the good with less loss of consumer surplus. On the other hand, the opposite is true for subsidies: a small, negative unit fee is more efficient than a small, proportional one. Consequently, under some circumstances, it can be optimal to tax under an A-agreement and subsidize under a U-agreement, but the reverse can never be true. Moreover, setting expression (8) to zero allows for confirmation that, when $\beta = \beta_A^M = \frac{p(q^M)/c}{\delta(q^M)}$, the platform has no incentive to deviate from $a = 0$.

We now state Corollary 2, characterizing the equilibrium quantities over the different ranges of β, making use of the following lemma.

Lemma 4. $\beta^0 < \beta_U^M$.

Corollary 2. *(a) If $\beta < \beta^0$, then $q_U < q_A < q_M$; (b) if $\beta \in \left(\beta^0, \beta_U^M\right)$, then $q_A < q_U < q_M$; (c) if $\beta \in \left(\beta_U^M, \beta_A^M\right)$, then $q_A < q_M < q_U$; (d) if $\beta > \beta_A^M$, then $q_M < q_A < q_U$.*

II.C Profits under U- and A-Agreements

This subsection shows that, when user access plays a significant role, U-agreements may be more profitable than A-agreements, both for the platform and the producer. Combining this finding with the results shown above in Subsection II.B, it then explores the potential (mis-)alignment among the various agents' preferences for either type of agreement. In particular, it identifies a range of β under which that the standard finding on the superiority of proportional fees can be completely reversed.

Proposition 3 compares the profits that arise under the two types of agreement, using the following definition. We denote by $\Pi_i(q)$ the platform's profits, under regime $i = U, A$, *from selling the good only*, and we denote its total profits by $\widehat{\Pi}_i(q)$=$\Pi_i(q) + \beta s(q)$. Recall, as well, that the $\pi_i(q)$ denotes the producer's profits.

Definition 5. *Let $\beta^{\widehat{\Pi}}$ denote a value of β that is* profit-neutral *for the platform. That is, if $\beta = \beta^{\widehat{\Pi}}$, the platform earns equal total profits under the two forms of agreement, i.e., $\widehat{\Pi}_U\left(q_U\right) = \widehat{\Pi}_A\left(q_A\right)$.*

Proposition 3. *(a) There exists a unique $\beta^{\widehat{\Pi}}$, and it lies in the interval $\left(\beta_U^M, \beta_A^M\right)$. Moreover, $\beta > \beta^{\widehat{\Pi}} \Leftrightarrow$ $\widehat{\Pi}_U(q_U) > \widehat{\Pi}_A(q_A)$.*

(b) $\beta \in \left[\beta^0, \beta_A^M\right] \Rightarrow \pi_U(q_U) > \pi_A(q_A)$.
Therefore, whenever $\beta \in \left(\beta^{\widehat{\Pi}}, \beta_A^M\right]$, switching from an A-agreement to a U-agreement increases the profits of the platform and the producer and raises consumer surplus ($s(q_U) > s(q_A)$).

The comparison of platform profits in part (a) stems from the following logic. If, under an agreement of form i, it is optimal for the platform to set a fee of zero (that is, if $\beta = \beta_i^M$), then it must earn strictly greater profits under the other form of agreement, in which it *does* strictly tax or subsidize. To see this, consider, for example, the case where the platform's optimal fee under an A-agreement is $a = 0$. The platform's profit would be unaffected by a switch to a U-agreement with r exogenously set to 0. Therefore, by optimally adjusting to $r \neq 0$, the platform necessarily boosts its profits. Symmetric logic applies to the case where the platform's optimal fee under a U-agreement equals 0.

Similar reasoning establishes that, for $\beta < \beta_U^M$, an A-agreement is more profitable for the platform, whereas, for $\beta > \beta_A^M$, a U-agreement is. To see this, note that, under the two forms of agreement, the platform's profits from sales of the good satisfy $\Pi_A(q) = \frac{p(q)}{mr(q)}\Pi_U(q)$ and that, for all relevant values of q, $p(q)/mr(q) > 1$. Thus, on the one hand, when β is small enough so that, regardless of the agreement, the platform taxes, it holds that $\widehat{\Pi}_A(q_U) > \widehat{\Pi}_U(q_U)$, and, therefore, *a fortiori*, $\widehat{\Pi}_A(q_A) > \widehat{\Pi}_U(q_U)$. On the other hand, when β is large enough so that, regardless of the agreement, the platform subsidizes the good, profits from sales of the good are negative. Analogous logic then implies that total profits are greater under a U-agreement. The proof of the existence of a unique $\beta^{\widehat{\Pi}}$ follows from these observations as well as an envelope theorem argument that guarantees a single crossing of the respective total profits under the two regimes.

Regarding the comparison in part (b), it is helpful to note that the producer's profits under the two forms of agreement satisfy $\pi_A(q) = \frac{c}{mr(q)}\pi_U(q)$. Thus, for any output level $q < q^M$ that is held fixed across the two arrangements, since $mr(q) < c$, it holds that $\pi_A(q) < \pi_U(q)$. That is, if a and r are set to a pair of nonnegative values that induce the producer to produce the same amount, under a U-agreement it retains a higher share of the profits than it does under an A-agreement. Keeping this in mind, the result then follows from considering these two cases. First, if $\beta \in \left[\beta^0, \beta_U^M\right]$, then the equilibrium values of a and r are indeed both nonnegative. Moreover, since $q_A < q_U$, the producer faces a lower effective marginal cost under a U-agreement, which further raises its profits. Second, if $\beta \in \left(\beta_U^M, \beta_A^M\right]$, then the platform taxes the good under an A-agreement and subsidizes it under a U-agreement, and so the latter is clearly preferable for the producer.

Note that, when the role of user access is either very small ($\beta < \beta^0$) or very large ($\beta > \beta_A^M$), it is ambiguous which form of agreement is more profitable for the producer. In the former case, as discussed above, for a fixed $q < q^M$, a U-agreement is more profitable. However, as $q_A > q_U$, the producer faces a higher effective marginal cost under a U-agreement, thus pushing in favor of an A-agreement. In the latter case, both forms of agreement feature subsidies, and for a fixed $q > q^M$, an A-agreement is preferable. However, as $q_U > q_A$, a U-agreement gives the producer a lower effective marginal cost, thus pushing in its favor.

For further illustration of Propositions 2 and 3, recall the two examples introduced above.

Example 1 (continued). *Under the constant elasticity specification, $p(q) = \alpha q^{-1/\varepsilon}$, where $\alpha > 0$ and $\varepsilon > 1$, the no-markup levels are $\beta_U^M = 1 - 1/\varepsilon$ and $\beta_A^M = 1$. To derive these, note that $\delta = \varepsilon/(\varepsilon - 1) = p(q)/mr(q)$, $\forall q$.*

Example 2 (continued). *Under the log-concave constant pass-through specification, $p(q) = \alpha - \theta q^{\mu}$, where $\alpha > c$, $\theta > 0$ and $\mu > 0$, the no-markup levels are $\beta_U^M = 1 + \mu$ and $\beta_A^M = 1 + \mu\alpha/c$. To derive these, note that $\delta = 1/(1 + \mu)$, and $p(q^M) = \frac{\mu\alpha + c}{\mu + 1}$.*

Figure 1 presents these two examples, side-by-side, taking curvature as the independent variable. Recall that, in the constant elasticity specification of Example 1, curvature lies in the interval $(1, 2)$, whereas Example 2's log-concave constant pass-through specification has curvature in $(0, 1)$. As the figure shows, in the more convex environment of Example 1, the main tension surrounds the issues dealt with in Propositions 2 and 3(a): does the platform tax or subsidize the good, and which form of agreement does it prefer? Here, the comparison of the good's price is a foregone conclusion with $p(q_U) < p(q_A)$, unless $\beta = 0$. Moreover, so long as $\beta \in [0, 1]$, the producer prefers a U-agreement. On the other hand, in the less convex environment of Example 2, the set of questions that are ambiguous is reversed. Either form of agreement may yield a lower good price, and it is not easy to guarantee that the producer prefers a U-agreement. Meanwhile, in this environment, unless $\beta > 1$, the platform taxes under both forms of agreement, and an A-agreement is always more profitable.

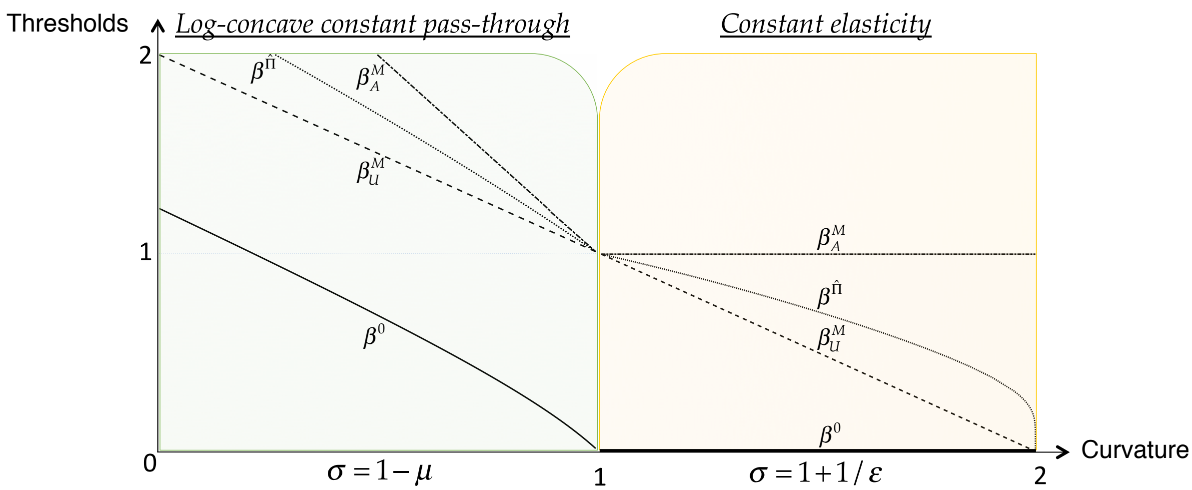

Figure 1: *Thresholds as functions of curvature in Examples 1 (right) and 2 (left).*[20]

Figure 2 summarizes the results stated above in this section. It helps to show two key points of the paper. The first point is that the preferences of the different parties for U- or A-agreements depend crucially on the role played by user access. The second point is that, under different circumstances, these preferences may or may not be aligned. In the figure, the top row shows the comparison, across the two forms of agreement, of the price of the good; the second row shows when the platform taxes or subsidizes it; the third row compares platform profits and the fourth row compares producer profits.

Prior literature comparing unit and proportional taxes/fees, beginning with Suits and Musgrave (1953), effectively ignores the potential role of user access and mainly highlights the benefits of A-agreements. However, as the figure shows, when user access plays a significant role, things change dramatically. On the one hand, when β is larger than β^0 and smaller than $\beta^{\widehat{\Pi}}$, the platform has an incentive to favor A-agreements over U-agreements, as in the standard case, even though this leads to a higher price for the good (due to greater double marginalization) and lower profits for the producer. On the other hand, when β exceeds $\beta^{\widehat{\Pi}}$, not only does the

[20]In Figure 1, we set $\alpha = 1$ and $c = 0.5$ for both examples and $\theta = 1$ for Example 2. In Example 2, $\beta^0 = 2\alpha\mu(1+\mu)^2 / \left[2\alpha\mu(1+\mu) + c + \sqrt{4\alpha\mu(1+\mu)c + c^2}\right]$.

platform prefer U-agreements, all parties may agree.

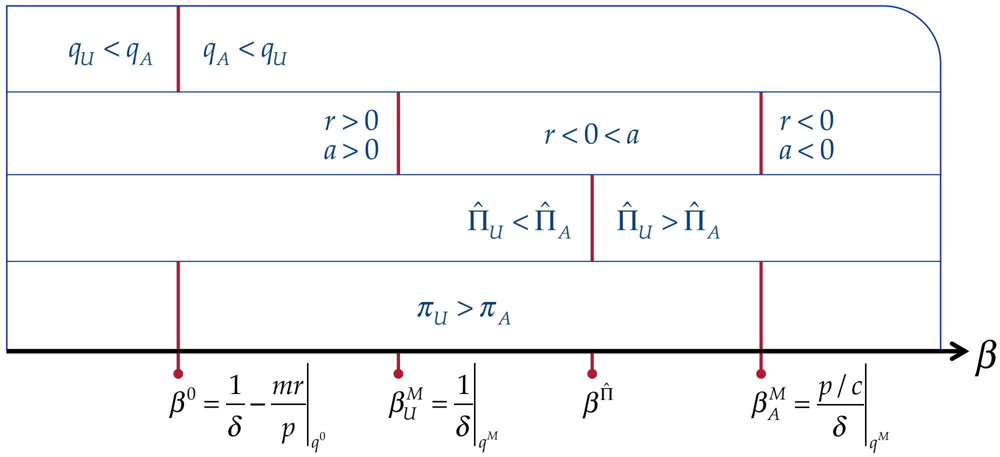

Figure 2: *Summary of Propositions 1 (top row), 2 (second row) and 3 (bottom rows).*

Of course, the approach taken in this section is quite stylized, as it considers just one dimension of the platform's profit maximization problem and takes user access, β, to be exogenous. The next section further develops the model by relaxing this simplification.

III Endogenous User Participation

This section extends the model of Section II. Here, the platform decides not only how much to tax or subsidize the good, but also how much revenue to extract directly from users by charging them for access. Specifically, we study the effects of varying the fraction of "dependent" users who must purchase access from the platform in order to be able to consume the good, and we show that, as this fraction increases, so does an endogenously determined variable that plays the same role as β in the previous section.

The new wrinkle is that, since the platform's optimal choice of how much to charge for user access depends on the form of agreement, this endogenous variable takes on different values across the two arrangements. Nevertheless, the main themes identified above continue to hold: as more of the potential good-buyers are captive to the platform, its incentives are pushed towards subsidizing. As this shift takes place, U-agreements become more desirable, first for users and then for the platform as well.

III.A Setup

There is a unit mass of consumers with quasilinear utility. A share $\gamma \in [0,1]$ of these are *dependent* users in that they must purchase access from the platform in order to buy and enjoy the good. The remaining $1-\gamma$ users are *autonomous*; that is, they have the ability to purchase the good using the platform without the need to purchase access. For example, in the electronic-book case, dependent users must purchase a device sold by the platform in order to access the platform's ebook "ecosystem," whereas autonomous users have the technical capacity to access the same ebook market directly by using, for example, their own smartphone or laptop.

Dependent users are heterogeneous in their "standalone" valuation, ξ, for access to the platform. For example, technology enthusiasts who place more intrinsic value in owning a new gadget have a high value of ξ, whereas, for others who prefer not to have another device in their lives, this variable could be negative. We assume that ξ is continuously distributed with cumulative distribution $F(\cdot)$.

Conditional on having access to the platform, all users are assumed to have identical demand for the good, given by $q(p)$. This function is assumed to have the same properties as detailed in Section II. Note, however, that here we are more restrictive than before in this regard. Whereas, in that section, this function merely described aggregate demand for the good, here it describes the demand of each individual. The various functions defined in Section II that stem from $q(p)$ retain their earlier definitions and economic meanings, with the caveat that their argument, q, denotes the quantity demanded by each individual.[21] In addition, we continue to make use of the array of objects defined in Section II, including output levels q^M (Def. 2) and q^0 (Lemma 2) as well as thresholds β^0 (Def. 3, *quantity-neutrality*), β_U^M and β_A^M (Def. 4, *no-markup*), that derive exclusively from the demand function and the producer's marginal cost, c.

Autonomous users care only about the good's price, p, purchasing $q(p)$ units. Dependent users, on the other hand, care about both p and t, which denotes the price that the platform charges for user access. A dependent user of type ξ joins the platform if and only if $\xi + s(q(p)) \geq t$. This says that, in deciding whether to purchase access, the user compares the sum of (i) her

[21]Specifically, recall: *inverse demand*, $p(q)$; *marginal revenue*, $mr(q)$; *consumer surplus*, $s(q)$; *price-elasticity*, $\varepsilon(q)$; *curvature*, $\sigma(q)$, and *pass-through*, $\delta(q)$.

standalone value from access and (ii) her anticipated (net) surplus from purchasing the good with (iii) the access price. Let $x \in [0,1]$ denote the share of dependent users who join the platform, and let $\widetilde{\xi}(x) \equiv F^{-1}(1-x)$ denote the marginal type, given x. We can then express inverse demand for user access as $t(q,x) = s(q) + \widetilde{\xi}(x)$.

The timing is as follows.

1. The platform sets access price t and, depending on the form of agreement, fee r or a.
2. Simultaneously: The producer sets the good's price, p; among the γ dependent users, γx purchase access and $\gamma(1-x)$ do not.
3. The $1-\gamma+\gamma x$ "eligible" users each purchase $q(p)$ units of the good.

Under this timing, the producer's optimal price is fully determined by the market for the good (i.e., it depends on marginal cost c, the platform's fee, r or a, and the demand function $q(\cdot)$, which is identical for all users). The producer does not have the ability to influence the number of eligible users who join. Because of this feature, the platform's choice of the user access price, t, can equivalently be represented as a choice of x.[22]

The platform's profits can now be derived. As before, we assume that the producer incurs marginal costs $c > 0$ for each unit sold of the good. Given the fee set by the platform, the producer best-responds as in equations (1) and (3), respectively under the two forms of agreement. Note that this best-response function is independent of t. Consequently, the platform's per-user profits *from the good only* are $\Pi_U(q) = (mr(q) - c)\,q$, and $\Pi_A(q) = \frac{mr(q)-c}{mr(q)} p(q)\,q$.[23] To express the platform's total profits, $\widehat{\Pi}_i$, $i = U, A$, note that it sells (i) both access and the good to γx dependent users and (ii) just the good to $1-\gamma$ autonomous users. Denote by $k \geq 0$ its per-user

[22] An alternative timing would put dependent users' joining decision in the last stage, at the same time as all users' decision of how much of the good to purchase. Under that alternative, the distinction of whether, in the first stage, the platform chooses t or x becomes one of substance. There, if the platform chooses x, then the producer's incentives remain exactly as they are here. However, if instead the platform chooses t, then the producer's incentives become more complicated, because changes in its price influence the number of users who join the platform. The timing that we adopt seems like the most reasonable one, both for its simplicity and because it matches our intuition that, given a large platform's weight in the economy, a producer is likely to consider its user base to be fixed. Analogous timing appears, for instance, in the platform models of White (2013) and Edelman and Wright (2015).

[23] See expressions (2) and (4), respectively.

cost of providing access. Thus,

$$\widehat{\Pi}_i(q,x) = \gamma x\Big(\Pi_i(q) + \underbrace{s(q) + \widetilde{\xi}(x)}_{t(q,x)} - k\Big) + (1-\gamma)\Pi_i(q). \tag{9}$$

We assume these profit functions to be globally concave with unique, interior solutions[24] and provide the relevant second-order conditions in Appendix A.2.

III.B Preliminary Results

This subsection develops the basic machinery for endogenizing β, the level of importance played by user access. First, we highlight an aspect of the profit functions under the two forms of agreement, expressed in (9), that plays an important role. Recall that, under either U- or A-agreements, $\Pi_i(q^M) = 0$. That is, when the platform neither taxes nor subsidizes the good, all of its profits come only from the direct sale of access. Moreover, since dependent users' willingness to pay for access depends on the good's price but not the form of agreement, holding fixed q, their demand for access is invariant with the form of agreement. These features lead the following level of access provision, formalized by Definition 6, to be especially salient.

Definition 6. *Let* $x^M \equiv \operatorname{argmax}_x \left\{\left(t(q^M,x) - k\right)x\right\}$ *denote the optimal choice of a hypothetical independent seller of access, when the good is priced at the integrated monopoly level,* $p(q^M)$*.*

Second, we define a set of values of γ that will serve as key thresholds.

Definition 7. *Under an agreement of form* $i = U, A$*, let* γ_i^0 *and* γ_i^M *denote shares of dependent users that gives rise, respectively, to equilibrium quantities of the good* q^0 *and* q^M*.*

Note that, unlike β^0, for reasons that will become clear in Lemma 6, γ_i^0 has an agreement-specific subscript. Third, we state Lemma 5, establishing the monotonicity of q_i and x_i with respect to γ. As for the previous section, all proofs are relegated to Appendix B.

Lemma 5 (Monotonicity of Output and Participation)**.** $q_i'(\gamma)$, $x_i'(\gamma) > 0$, *for all* $\gamma \in (0,1)$, $i = U, A$. *That is, under both forms of agreement, the equilibrium (a) quantity of the good and (b) share of dependent*

[24]Of course, when $\gamma = 0$, the choice of x is irrelevant; in this case, for technical purposes, we assume that the platform sets $x_i = 0$, $i = U, A$.

users who purchase access are both strictly increasing functions of the share of dependent users.

This monotonicity can be understood using the following intuition. As the share of dependent users, γ, increases, the group whom the platform would optimally treat as "two-part tariff" customers grows, relative to the group it would treat as "linear pricing" customers. The larger weight of the former group leads to an incentive to tax the good less and thus a lower good price and a higher equilibrium value of q. Then, since the price of the good is lower, when they make their joining decisions, dependent users anticipate more net surplus, $s(q)$, from the good. In the platform's decision of how many users it sells access to, this increased net surplus is equivalent to a decrease in marginal cost, leading to an increase in x.

The following definition begins to build the bridge between the simplified approach of Section II and the current environment with endogenous user access.

Definition 8. *Let* $\beta(x,\gamma) \equiv \frac{\gamma x}{\gamma x+1-\gamma}$ *denote the* dependent share of purchasers *of the good.*

Note that, whereas γ is the share of *all* users who must buy access in order to consume the good, $\beta(x,\gamma)$ captures the share of dependent users among all eligible users. Lemma 6 now shows the mapping between the thresholds in γ and the key levels of β identified in Section II.

Lemma 6. *Under an agreement of form* $i = U, A$*,*

(a) there exists a unique $\gamma_i^0 \in [0,1]$ *if and only if* $\beta^0 \in [0,1]$*; moreover,* $\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) = \beta^0$*;*

(b) there exists a unique $\gamma_i^M \in (0,1]$ *if and only if* $\beta_i^M \leq 1$*; moreover,* $\beta\left(x_i(\gamma_i^M), \gamma_i^M\right) = \beta\left(x^M, \gamma_i^M\right) = \beta_i^M$*.*

If they both exist, then $\gamma_i^0 < \gamma_i^M$*.*

To understand Lemma 6, consider part (b) first. It says that, under an agreement of form i, whenever $\beta_i^M \leq 1$ (and thus lies in the unit interval) there exists a specific share of dependent users that induces the platform to neither tax nor subsidize the good. In this situation, since the platform makes zero profit from each unit sold of the good, the pricing problem for user access becomes a standard monopoly problem, and it serves fraction x^M of the dependent users.

In part (a), the endogenous choice of user access leads to a bit more subtlety. Here, under an agreement of form i, when $\gamma = \gamma_i^0$, the platform's optimal access choice, $x_i(\gamma_i^0)$, is such that γ_i^0

and $x_i(\gamma_i^0)$ jointly lead to $\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) = \beta^0$. At such an equilibrium, if, hypothetically, the form of agreement were to switch to $j \neq i$, but the platform's user access choice were exogenously held fixed at $x_i(\gamma_i^0)$, then it would have no incentive to deviate in the good market from q^0. However, if the form of agreement switched and the platform were to fully re-optimize, then it may have an incentive to adjust its action in both dimensions. Thus, under an agreement of form $j \neq i$, a different share of dependent users, $\gamma_j^0 \neq \gamma_i^0$ may give rise to $\beta\left(x_j(\gamma_j^0), \gamma_j^0\right) = \beta^0$.

Lemma 6 naturally suggests the following classification, established in Definition 9, regarding which good markets do or do not admit these critical levels, γ_i^0 and γ_i^M.

Definition 9. *The market for the good* $(q(p), c)$ *is*

(a) q^0-compatible *if* $\beta^0 \in [0, 1]$,

(b) iM-compatible *if* $\beta_i^M \leq 1$, $i = U, A$.

We say a market is FIT *(Full Interior Threshold) if it satisfies* (a) *and* (b), *for both forms of agreement.*

This definition regards the levels of importance for user access that a good market requires in order to exhibit the two types of neutrality discussed in the exogenous-β environment of Section II. (See Definitions 3 and 4.) Part (a) deals with quantity-neutrality across agreements, holding fixed β. The good market is q^0-compatible when the level, β^0, that causes the platform's optimal quantity in the good market to be unaffected by the form of agreement lies in the closed unit interval. Part (b) deals with the platform's optimal fee. That is, the good market is iM-compatible when the level, $\beta_i^M > 0$, that leads the platform to set a fee of zero under an agreement of type i is no greater than one.

In the current, endogenous setting, Lemma 6 thus implies: (a) whenever a good market is q^0-compatible, under an agreement of form $i = U, A$, there exists exactly one share of dependent users, γ_i^0, that induces $\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) = \beta^0$; (b) whenever a good market is iM-compatible, there exists exactly one share of dependent users that, under an agreement of form i, induces the platform to set a zero fee.

We now state Lemma 7, which is the final step necessary for making the economic comparisons of interest across the two forms of agreement. Note that AM-compatibility implies UM-compatibility, because $\beta_U^M < \beta_A^M$.

Lemma 7. *(a) In a q^0-compatible market, if $\beta^0 \in (0,1)$, then $\gamma_A^0 < \gamma_U^0$, and if $\beta^0 \in \{0,1\}$, then $\gamma_A^0 = \gamma_U^0 = \beta^0$.*

(b) In an AM-compatible market, $\gamma_U^M < \gamma_A^M$.

Note that markets with constant elasticity demand, as in Example 1, are FIT, whereas those with log-concave constant pass-through demand, as in Example 2, are not. Subsection III.C considers the former case, and then Subsection III.D considers the latter.

III.C Prices, Profits and Welfare in Markets with Interior Thresholds

We now turn to the main results under the endogenous provision of user access. This subsection restricts attention to FIT good markets, i.e., those with $0 \leq \beta^0 < \beta_U^M < \beta_A^M \leq 1$. Such markets provide an ideal setting for understanding the endogenous version of the model, because they encompass our entire array of comparison points between the two forms of agreement. In Subsection III.D, we move on to consider more general demand and show that the main insights derived here carry through.

III.C.1 Equilibrium Pricing

Proposition 4 states the conditions under which the platform taxes or subsidizes the good.

Proposition 4. *When $\gamma < \gamma_U^M$, the platform taxes under both forms of agreement ($r, a > 0$). When $\gamma \in (\gamma_U^M, \gamma_A^M)$, the platform subsidizes under a U-agreement and taxes under an A-agreement ($r < 0 < a$). When $\gamma > \gamma_A^M$, the platform subsidizes under both forms of agreement ($r, a < 0$).*

The configuration described in Proposition 4 mirrors the one described in Proposition 2. When the share of dependent users is sufficiently low, the platform taxes, and when this share is sufficiently high, the platform subsidizes the good. Moreover, the platform may subsidize under an A-agreement and tax under a U-agreement, but the opposite can never be true. The logic for why this result still holds under endogenous user access stems from Lemma 6, which shows that, at the threshold levels γ_i^M, $i = U, A$, the dependent share of eligible users satisfies $\beta\left(x_i(\gamma_i^M), \gamma_i^M\right) = \beta_i^M$, and that $x_i(\gamma_i^M) = x^M$. In other words, since, conditional on setting its sales fee to 0, the platform sells access to the same fraction of dependent users regardless of the form of agreement, and since $\beta_A^M > \beta_U^M$, the no-markup level of dependent users is higher under an A-agreement.

Proposition 5 now compares the ranking of prices and levels of access provision under the two forms of agreement, making use of Definition 10.

Definition 10. *If it exists, denote by γ^x the maximum value of $\gamma \in [0, 1]$ such that $x_U(\gamma) = x_A(\gamma)$.*

Proposition 5. *When the share of dependent users is small, an A-agreement gives rise to a weakly lower price of the good and a weakly larger share of dependent users who join the platform. However, when the share of dependent users is large, the good's price is strictly lower and strictly more users join under a U-agreement. Formally,*

(a) If $\gamma \in \left[0, \gamma_U^0\right]$, then $q_A \geq q_U$ and $x_A \geq x_U$. The first inequality is strict unless $\gamma_U^0 = 0$, and the second is strict unless $\gamma = 0$.

(b) γ^x exists and lies in the interval $\left(\gamma_U^0, \gamma_U^M\right)$. If $\gamma \in [\gamma^x, 1]$, then $q_U > q_A$ and $x_U \geq x_A$, where the second inequality is strict unless $\gamma = \gamma^x$.

Therefore, in case (a), all users are weakly better off under an A-agreement, whereas, in case (b), all users are weakly better off under a U-agreement.

Proposition 5 shows that, when the share of dependent users is sufficiently low or sufficiently high, endogenous provision of user access reinforces the price comparison results of Proposition 1. Intuitively speaking, part (a) says that, starting from equilibrium under a U-agreement, switching to an A-agreement leads to not only a direct incentive to increase output of the good, as shown in Proposition 1, but also it increases the marginal benefit of adding users. This is true in this setting, because the initially posited equilibrium involves taxing, and for a given output level of the good, $q < q^M$, profits per user from sales of the good alone are greater under an A-agreement: $\Pi_A(q) > \Pi_U(q)$. Moreover, the two effects feed into one another, because, as the number of dependent users increases, the platform's optimal pricing strategy shifts more towards a two-part tariff approach and away from a linear pricing approach.

Part (b) appeals to two different arguments depending on γ. If $\gamma_A^M < 1$ and $\gamma \in \left(\gamma_A^M, 1\right]$, then a symmetric argument to the one used in part (a) applies. It says that, since an initially posited

A-agreement involves subsidy, switching to a U-agreement gives the platform a direct incentive to add users. Furthermore, this incentive to add users complements the direct incentive, shown in Proposition 1, to lower the good's price. On the other hand, if $\gamma \in \left[\gamma^x, \gamma_A^M\right]$, since $\gamma_U^M < \gamma_A^M$, yet these two shares of dependent users lead to identical outcomes under the respective agreements, the argument follows from the monotonicity shown in Lemma 5.

III.C.2 Profits and Welfare

Definition 11. *Let $\gamma^{\widehat{\Pi}}$ denote a* profit-neutral *share of dependent users. That is, if $\gamma = \gamma^{\widehat{\Pi}}$, the platform earns equal total profits under the two forms of agreement, i.e., $\widehat{\Pi}_U(q_U, x_U) = \widehat{\Pi}_A(q_A, x_A)$.*

Note that, in this section's setup, $\pi_i(q)$, first defined in Lemma 1, denotes the producer's profits *per purchaser of the good*, under an agreement of form $i = U, A$. The producer's total profits are now given by $\hat{\pi}_i(q, x) = (\gamma x + 1 - \gamma)\pi_i(q)$.

Proposition 6. *(a) There exists a unique $\gamma^{\widehat{\Pi}}$, and it lies in the interval $\left(\gamma_U^M, \gamma_A^M\right)$. Moreover, $\gamma > \gamma^{\widehat{\Pi}} \Leftrightarrow \widehat{\Pi}_U(q_U, x_U) > \widehat{\Pi}_A(q_A, x_A)$.*

(b) $\gamma \in \left[\gamma^x, \gamma_A^M\right] \Rightarrow \hat{\pi}_U(q_U) > \hat{\pi}_A(q_A)$.

A key point of Proposition 6 is that, when enough of the users are of the dependent type, both the platform and the producer prefer a U-agreement. In particular, part (a) extends Proposition 3(a)'s finding on the relationship between the platform's choice to tax or subsidize the good and its preference over agreements. Part 6(b) extends Proposition 3(b), regarding the producer's preference for U-agreements.[25]

The intuition for why part (a) still holds under endogenous provision of user access is as follows. Under a U-agreement, consider an equilibrium featuring (q_U, x_U). Now, suppose that the arrangement switches to an A-agreement but that the platform must hold fixed x_U and can re-optimize only in the dimension of q. If, at the initial equilibrium, the platform chooses to tax the good, then, according to Proposition 3, upon switching to an A-agreement and re-optimizing with respect to q, the platform's profits will increase. Therefore, if, after switching

[25] Although we can prove the producer's preference for a U-agreement only for the interval stated in Proposition 6(b), we know of no example, with γ outside this interval, where the producer prefers an A-agreement.

to an A-agreement, the platform could re-optimize in both dimensions, it must *a fortiori* enjoy an increase in profits. Likewise, consider an equilibrium under an A-agreement in which the platform subsidizes the good. If the regime switches to a U-agreement, and the platform can re-optimize only with respect to q, then, according to Proposition 3, its profits will increase. Therefore, *a fortiori*, if it can re-optimize in both dimensions, its profits must increase.

We can now state Proposition 7, which also takes into account users.

Proposition 7. *Whenever* $\gamma \in \left[\gamma^{\widehat{\Pi}}, \gamma_A^M\right]$*, a U-agreement Pareto-dominates an A-agreement.*

This result fully captures the idea, mentioned informally at the end of Section II, regarding the possible alignment among the agents in preferring a U-agreement. Here, unlike in that setting, we explicitly model user preferences and are thus are able to compare the payoffs of all types of agents across the two forms of agreement. This is the first result in the literature, of which we are aware, showing the possibility of such an alignment, in favor of U-agreements, among the platform/tax authority, a seller, and consumers. It specifically reverses Shy and Wang's (2011) comparisons regarding platform profits and consumer surplus across the two forms of agreement. In a rougher sense, it can be seen as a "mirror image" of Suits and Musgrave's (1953) and Skeath and Trandel's (1994) respective arguments for the welfare dominance and Pareto optimality of *ad valorem* commodity taxes, compared to to unit taxes.[26]

To see why, when $q_U > q_A$ and $x_U > x_A$, all users are weakly better off under a U-agreement, consider the two categories. The $1-\gamma$ autonomous users purchase only the good and thus do so at a lower price under a U-agreement. Regarding the γ dependent users, although it is possible that the equilibrium price for access may be higher, since $x_U > x_A$, the decrease in the price of the good is sufficiently large so that the type who would be marginal under an A-agreement strictly prefers to join under a U-agreement. Moreover, since dependent users are homogeneous in their demand for the good, they all agree that the overall deal offered by the platform under a U-agreement more appealing.

In summary, this subsection, which assumes FIT demand, reinforces one of the main messages of Section II, that the comparison between U- and A-agreements depends crucially on

[26]The comparison between our results to those of Shy and Wang (2011) is more direct than it is to those of Suits and Musgrave (1953) and Skeath and Trandel (1994), because the former assume that the platform maximizes its profits, whereas the latter assume that the tax authority must satisfy a minimum revenue constraint.

user access. Taking the endogenous provision of user access into account makes the analysis technically more involved, but, as we discuss following each of the propositions, the additional dimension largely enhances the effects that govern when β is taken to be exogenous.[27] By restricting attention to FIT demand, which, recall, includes the constant elasticity specification of Example 1, we guarantee that all of the relevant thresholds exist, but we ignore the role of variation in demand curvature. We turn to this latter issue in the next subsection.

III.D Markets with More General Demand

This subsection describes how the results from Subsection III.C can be extended or modified when demand for the good falls into each of three broad classes, using Mràzovà and Neary's (2017) partition. First it covers the two most straightforward cases of strictly log-concave demand and super-convex demand. Then it addresses the middle class of demand that is log-convex yet sub-convex. Note that constant elasticity demand lies on the boundary between the latter two classes. Appendix C.3 formally derives all of the claims made in this subsection.

Strictly Log-Concave Demand. When demand is globally strictly log-concave, it is neither *AM*- nor *UM*-compatible. Therefore, the platform subsidizes the good under both arrangements, and it always prefers an A-agreement. If demand is not too concave, then $\beta^0 \leq 1$, and the market is q^0-compatible. In this case, the claim in Proposition 5(a) applies: that is, if the share of dependent users is below γ_U^0, then $q_A > q_U$ and $x_A > x_U$. If the market is not q^0-compatible, then these inequalities holds regardless of the share of dependent users.

Super-Convex Demand. When demand is globally strictly super-convex, the good market is both *AM*- and *UM*-compatible, but it is not q^0-compatible. Regarding both the platform's choice of whether to tax/subsidize the good and its preference over the two forms of agreement, the same pattern applies as in a FIT market. Moreover, like a FIT market, when γ exceeds a lower bound (either γ^x or 0), $q_U > q_A$ and $x_U > x_A$. Unlike a FIT market, no value of $\gamma \geq 0$ is sufficiently small to guarantee the outcome in which these inequalities are reversed.

[27] The main caveat to this claim involves the interval (γ_U^0, γ^x), in which the platform's direct incentives in the two dimensions conflict in a way that makes general comparisons between equilibrium values of q's and x's infeasible.

Log-Convex, Sub-Convex Demand. When demand is, globally, both strictly log-convex and strictly sub-convex, it is q^0-compatible and UM-compatible but not AM-compatible. When $\gamma < \gamma_U^M$, the platform taxes under both arrangements; otherwise it subsidizes under a U-agreement and taxes under an A-agreement. Regarding the remaining issues, the key factor that determines the extent to which a market in this category resembles a FIT market is whether $\beta^{\widehat{\Pi}} \leq 1$ or not. This inequality holds in markets closer to the boundary with super-convexity, but it is violated in markets closer to the boundary with log-concavity.[28] When this inequality holds, versions of Propositions 5, 6, and 7 all hold, with the slight modification that it is no longer possible to rank γ^x and γ_U^M.

On the other hand, when $\beta^{\widehat{\Pi}} > 1$, the platform prefers an A-agreement no matter the share of dependent users. Furthermore, Proposition 5(a) continues to hold. However, for large shares of dependent users, it is not possible to guarantee that users prefer a U-agreement over an A-agreement.

Broadly speaking, these results on the role of convexity match those of Section II. The more convex demand is, the less stringent the conditions on the share of dependent users become in order for (a) the platform to switch from a tax to a subsidy, (b) all agents, including the platform, to prefer a U-agreement.

IV Conclusion

We compare two types of transaction fee that platforms charge sellers: per-unit and proportional. Our key motivation is the fact that, to varying degrees, platforms also charge their users for "access" via markups on hardware and/or subscription fees. For example, Amazon tries to break even on its Kindle hardware to help it earn profits selling digital media. On the other hand, Apple took the opposite approach in selling iPods and digital music.

A long line of earlier work (e.g., Daniel B. Suits and Richard A. Musgrave, 1953; Robert L. Bishop, 1968; Susan E. Skeath and Gregory A. Trandel, 1994; Simon P. Anderson, Andre de Palma and Brent Kreider, 2001*a*,*b*; Oz Shy and Zhu Wang, 2011; Justin P. Johnson, 2017) studying these two types of pricing arrangements holds that proportional fees typically are more efficient in

[28] See Example 3 in Appendix C.4 for an illustration.

that they lead to lower good prices and higher profits for the platform (or revenues for a tax authority). We show that, when user access plays a significant role, per-unit fees can become more efficient.

More specifically, in a standard setting without regard for user access, as long as demand is not extremely convex, proportional fees are more efficient. We develop a model showing, in effect, that this "convexity threshold" tightens as the platform's user access fee grows. When the platform charges a high user access fee, demand need not be very convex in order for per-unit fees to be more efficient. We also show that, whereas it may be optimal to "tax" the sale of a good under a proportional fee regime and to "subsidize" it under a per-unit regime, the opposite is never true; moreover, we characterize the relationship between this issue and the profitability of the two forms of agreement.

In the world of powerful digital platforms, our results may have significant bearing on policy, because they bring to light previously unappreciated aspects of two types of vertical agreement that such platforms commonly use. As we discuss, the antitrust case in the electronic-book industry is one situation where taking user access into account appears particularly salient in interpreting what occurred. However, the basic issues at the heart of the paper also arises in the context of payment cards, online travel, app stores, etc.

Of course, the current paper's model has some limitations. For example, the model presented in Section II considers fully general demand for the usage good, including allowing for variation across users in how much they purchase. However, in Section III, when we endogenize the platform's sale of user access, we restrict user heterogeneity to the one dimension of their participation value. Incorporating richer heterogeneity in a tractable way, although technically challenging, could be an exciting avenue for future research.

Appendices

A Second-Order Conditions

A.1 Conditions Applying to Section II

The following conditions apply in Section II. First, Condition 1, from standard monopoly theory, guarantees that, under both forms of agreement, the producer's first-order condition is sufficient for maximization.

Condition 1. *$mr'(q) < 0$ for all q such that $mr(q) > 0$ (and also for the limit as q tends to 0).*

Second, let $U(q) \equiv \beta s'(q) + mr(q) + q\, mr'(q)$. Condition 2 guarantees that, under a U-agreement, the platform's first-order condition is sufficient for maximization.

Condition 2. *$U'(q) < 0$ for all q such that $U(q) > 0$.*

Third, let $A(q) \equiv (\beta s'(q) + mr(q)) / \left(1 - q\, mr'(q)\, p(q) / mr(q)^2\right)$. Condition 3 guarantees that, under an A-agreement, the platform's first-order condition is sufficient for maximization.

Condition 3. *$A'(q) < 0$ for all q such that $A(q) > 0$.*

A.2 Conditions Applying to Section III

The following conditions apply in Section III. First, we continue to impose Condition 1, guaranteeing that, with respect to the sale of the good, the producer's first-order condition is sufficient for maximization under both forms of agreement.

Second, Condition 4 guarantees that when $\gamma \in (0,1]$, at equilibrium, the platform sells access to an interior share, $x \in (0,1)$, of dependent users. Let $\overline{q}_i \equiv \operatorname{argmax}_q \left\{\widehat{\Pi}_i(q,x)|_{\gamma=1}\right\}$, where $\left.\frac{\partial \widehat{\Pi}_i}{\partial q}\right|_{\gamma=1} = \Pi_i'(q) + s'(q)$.

Condition 4. *For all $\gamma \in (0,1]$ and for all $q \in (0, \overline{q}_i)$, (i) $\lim_{x\to 0} \frac{\partial \widehat{\Pi}_i}{\partial x} > 0$, and (ii) $\lim_{x\to 1} \frac{\partial \widehat{\Pi}_i}{\partial x} < 0$, $i = U, A$.*

Given the definition of $\widehat{\Pi}_i$, parts (i) and (ii) of Condition 4 are equivalent to $\lim_{x\to 0} \Pi_i(q) + s(q) + \widetilde{\xi}(x) + x\,\widetilde{\xi}'(x) - k > 0$ and $\lim_{x\to 1} \Pi_i(q) + s(q) + \widetilde{\xi}(x) + x\,\widetilde{\xi}'(x) - k < 0$, respectively.

Third, Condition 5 guarantees that, under both forms of agreement, the platform's first-order conditions are sufficient for maximization.

Condition 5. *For all $\gamma \in [0,1]$, (i) $\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2} < 0$, (ii) $\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} < 0$, and (iii) $\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2} \frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} > \left(\frac{\partial^2 \widehat{\Pi}_i}{\partial q \partial x}\right)^2$, $i = U, A$.*

Condition 5 implies that $\widehat{\Pi}_i$ is globally strictly concave, and optimal choices of q_i and x_i are unique, $i = U, A$. Given the definition of $\widehat{\Pi}_i$, parts (i) and (ii) of Condition 5 are equivalent to $[1 - \gamma + x\gamma]\, \Pi_i''(q) + x\gamma\, s''(q) < 0$ and $2\,\widetilde{\xi}'(x) + x\,\widetilde{\xi}''(x) < 0$, respectively. The cross-partial term in part (iii) of Condition 5, $\frac{\partial^2 \widehat{\Pi}_i}{\partial q \partial x}$, is equal to $\gamma\left(\Pi_i'(q) + s'(q)\right)$.

B Proofs

Proof of Lemma 1. Under both U- and A-agreements, the total profits of the platform and the producer from sales of the good are equal to $(p(q) - c)\,q$. Therefore, for $i = U, A$, we can rewrite the platform's objective function as

$$\max_q \,(p(q) - c)\,q - \pi_i(q) + \beta s(q)\,. \tag{10}$$

The first-order condition of the optimization problem (10) is

$$mr(q_i) - c - \pi_i'(q_i) + \beta\, s'(q_i) = 0. \tag{11}$$

Hence, it suffices to show that $\pi_U'(q) = -q\, mr'(q)$ and $\pi_A'(q) = -q\, mr'(q)\,\frac{p(q)/mr(q)}{mr(q)/c}$.

Substitute (1) into the producer's profits under a U-agreement, $[p(q) - (c + r)]\,q$. We have $\pi_U(q) = (p(q) - mr(q))\,q = p(q)\,q - mr(q)\,q$. Note that $mr(q) = p(q) + q\,p'(q) = (p(q)\,q)'$. Thus $\pi_U'(q) = mr(q) - q\,mr'(q) - mr(q) = -q\,mr'(q)$.

Substitute (3) into the producer's profits under an A-agreement, $[(1-a)\,p(q) - c]\,q$. We have

$$\pi_A(q) = \left(\frac{c}{mr(q)}p(q) - c\right)q = \left(\frac{p(q)}{mr(q)}q - q\right)c.$$

Note that

$$\begin{aligned}\left(\frac{p(q)}{mr(q)}q\right)' &= \frac{p(q)}{mr(q)} + q\left(\frac{p(q)}{mr(q)}\right)' = \frac{p(q)}{mr(q)} + q\,\frac{p'(q)\,mr(q) - p(q)\,mr'(q)}{mr(q)^2}\\ &= \frac{(p(q) + q\,p'(q))\,mr(q) - q\,p(q)\,mr'(q)}{mr(q)^2} = 1 - \frac{q\,p(q)\,mr'(q)}{mr(q)^2}.\end{aligned}$$

Thus

$$\pi_A'(q) = \left(1 - \frac{q\,p(q)\,mr'(q)}{mr(q)^2} - 1\right)c = -q\,mr'(q)\,\frac{p(q)\,/mr(q)}{mr(q)\,/c}.$$

□

Proof of Lemma 2. $\frac{c\,p(0)}{mr(0)^2} < 1$, because either (i) $p(0)$ is well defined (and, hence, so is $mr(0) = p(0)$), and $c < p(0)$, or (ii) both $\lim_{q\to 0} p(q) = +\infty$ and $\lim_{q\to 0} mr(q) = +\infty$ and, applying L'Hospital's rule and using $\lim_{q\to 0}\sigma(q) < 2$, $\forall q$, from the second-order conditions, we obtain $\lim_{q\to 0} c\,p(q)/mr(q)^2 = 0$. Also, we have $\frac{c\,p(q^M)}{mr(q^M)^2} = \frac{p(q^M)}{mr(q^M)} > 1$. Therefore, given Assumption 1, there exists a unique $q^0 \in (0, q^M)$ such that $\frac{c\,p(q^0)}{mr(q^0)^2} = 1$. Moreover, $\forall q > q^M$ such that $mr(q) > 0$, $p(q)/mr(q) > mr(q)/c$ holds, because $p(q)/mr(q) > 1 > mr(q)/c$ throughout this interval. □

Proof of Proposition 1. We first show that when $\beta < \beta^0$, we have $q_A > q_U$. If $mr(q^0) - c < -\beta s'(q^0) + \pi_U'(q^0)$, then we must have $q_U, q_A < q^0$. Since for $q < q^0$, $\pi_A'(q) < \pi_U'(q)$, from Lemma

2, it must be that $q_U < q_A < q^0$. As $c = \frac{mr^2(q^0)}{p(q^0)}$, $mr(q^0) - c < -\beta s'(q^0) + \pi'_U(q^0)$ amounts to $\beta < \frac{\pi'_U(q^0)}{s'(q^0)} - \frac{mr(q^0)-c}{s'(q^0)} = \frac{mr'(q^0)}{p'(q^0)} - \frac{mr(q^0)}{p(q^0)} \frac{p(q^0)-mr(q^0)}{-q^0 p'(q^0)} = \frac{-q^0 \varepsilon'(q^0)}{\varepsilon(q^0)} \equiv \beta^0$.

Reversing inequalities, an analogous argument shows that, when $\beta > \beta^0$, we have $q_U > q_A > q^0$. The case of equality follows in a straightforward manner. □

Proof of Corollary 1. See Proposition 1, with $\beta = 0$. □

Proof of Lemma 3. Part (a) is straightforward, since global strict log-concavity (convexity) of demand implies that, for all q, $\delta(q) < 1$ ($\delta(q) > 1$). Part (b) holds because, for all q, $\varepsilon'(q) < 0 \Leftrightarrow \frac{p(q)/mr(q)}{\delta(q)} > 1$, and $mr(q^M) = c$. □

Proof of Proposition 2. Under a U-agreement, the platform's first-order condition is $mr(q_U) - c + q_U\, mr'(q_U) + \beta s'(q_U) = 0 \Leftrightarrow \beta = \frac{1}{\delta(q_U)} - \frac{mr(q_U)-c}{-q_U\, p'(q_U)}$. Therefore, if $\beta = \beta_U^M$, it must be that $q_U = q^M \Leftrightarrow mr(q_U) = c \Leftrightarrow r = 0$; that is, the platform neither taxes nor subsidizes the good, and the producer acts as though it were a monopolist. Furthermore, since $s'(q) > 0$ for all q, the implicit function theorem can be used to show that q_U strictly increases with β. Hence, $\beta < \beta_U^M \Leftrightarrow q_U < q_M \Leftrightarrow mr(q_U) > c \Leftrightarrow r > 0$.

Under an A-agreement, the platform's first-order condition is $mr(q_A) - c + q_A\, mr'(q_A) \frac{p(q_A)/mr(q_A)}{mr(q_A)/c} + \beta s'(q_A) = 0 \Leftrightarrow \beta = \frac{p(q_A)/mr(q_A)}{\delta(q_A)} \left(\frac{c}{mr(q_A)}\right)^2 - \frac{mr(q_A)-c}{-q_A\, p'(q_A)}$. Hence, if $\beta = \beta_A^M$, then $q_A = q^M \Leftrightarrow mr(q_A) = c \Leftrightarrow a = 0$. Furthermore, since q_A strictly increases with β, we have $\beta < \beta_A^M \Leftrightarrow q_A < q_M \Leftrightarrow mr(q_A) > c \Leftrightarrow a > 0$. □

Proof of Lemma 4. The second-order conditions imply that $q'_i(\beta) > 0$, and Lemma 2 shows that $q^0 < q^M$. □

Proof of Corollary 2. Part (a) follows directly from Proposition 1 ($\beta < \beta^0 \Rightarrow q_U < q_A < q^0$) and the fact that $q^0 < q^M$. For parts (b), (c) and (d): first, Proposition 1 also shows that $\beta > \beta^0 \Rightarrow q_A < q_U$. In all three parts, the antecedent is satisfied, and, thus, the conclusion is as well. Second, the comparisons to q_M follow directly from Proposition 2. □

Proof of Proposition 3. To prove part (a), we show that (i) $\beta \le \beta_U^M \Rightarrow \widehat{\Pi}_A(q_A) > \widehat{\Pi}_U(q_U)$, (ii) $\beta \ge \beta_A^M \Rightarrow \widehat{\Pi}_U(q_U) > \widehat{\Pi}_A(q_A)$, and (iii) for all $\beta \in \left(\beta_U^M, \beta_A^M\right)$, it holds that $\frac{d}{d\beta}\left\{\widehat{\Pi}_U(q_U)\right\} > \frac{d}{d\beta}\left\{\widehat{\Pi}_A(q_A)\right\}$.

(i) Expressions (2) and (4) imply that the following equation holds, relating the platform's profits, under the two forms of agreement, derived only from selling the good:

$$\Pi_A(q) = \frac{p(q)}{mr(q)}\Pi_U(q), \tag{12}$$

where $\Pi_U(q) = (mr(q) - c)q$. Thus, for any $\hat{q} \in \left(0, q^M\right)$, since $p(\hat{q}) > mr(\hat{q}) > c$, we have $\widehat{\Pi}_A(\hat{q}) > \widehat{\Pi}_U(\hat{q})$. Moreover, $\beta < \beta_U^M$, and thus $q_U \in \left(0, q^M\right)$. Therefore, $\widehat{\Pi}_U(q_U) < \widehat{\Pi}_A(q_U) < \widehat{\Pi}_A(q_A)$.

That is, holding fixed the equilibrium quantity q_U and switching from a U-agreement to an A-agreement increases the platform's total profits; furthermore, under an A-agreement, q_A is optimal for the platform, whereas q_U is not.

(ii) Following similar logic, since $\beta > \beta_A^M$, we have $q_A > q^M$. Moreover, Condition 3 in Appendix A guarantees that q_A is sufficiently small so that $mr(q_A) > 0$. Therefore, since $c > mr(q_A) > 0$, we have $\widehat{\Pi}_A(q_A) < \widehat{\Pi}_U(q_A) < \widehat{\Pi}_U(q_U)$.

(iii) We now express the platform's profits under an agreement of form $i = U, A$ as implicit functions of β, i.e., $\widehat{\Pi}_i\left(q_i(\beta), \beta\right)$. Totally differentiating with respect to β, the envelope theorem implies

$$\frac{d}{d\beta}\left\{\widehat{\Pi}_i\left(q_i(\beta), \beta\right)\right\} = \underbrace{\frac{\partial \widehat{\Pi}_i}{\partial q}}_{0} \cdot q_i'(\beta) \;+\; \frac{\partial \widehat{\Pi}_i}{\partial \beta} \;=\; s\left(q_i(\beta)\right).$$

Note that $s(\cdot)$ is a strictly increasing, continuous function. Since $\beta_U^M > \beta^0$, Proposition 1 implies that $q_U(\beta) > q_A(\beta)$ over the interval $\left(\beta_U^M, \beta_A^M\right)$. Therefore, over this interval, $\frac{d}{d\beta}\left\{\widehat{\Pi}_U(q_U)\right\} > \frac{d}{d\beta}\left\{\widehat{\Pi}_A(q_A)\right\}$.

Together, (i), (ii) and (iii) establish the claims made in part (a).

To prove part (b), first note that, under an agreement of form $i = U, A$, the producer's profits are given by $\pi_i(q) = (p(q) - c)q - \Pi_i(q)$. Further note that $\pi_i(\cdot)$ are strictly increasing functions, because an increase in q is tantamount to a decrease in the producer's marginal cost. Plugging in for $\Pi_i(q)$ and simplifying gives $\pi_u(q) = qs'(q)$, and $\pi_A(q) = \frac{c}{mr(q)} qs'(q)$. Thus, for all $q \in \left(0, q^M\right]$, it holds that $\pi_U(q) \geq \pi_A(q)$. Since $\beta \leq \beta_A^M$, we have $q_A \leq q_M$, and since $\beta > \beta^0$, we have $q_A < q_U$. Therefore, $\pi_A(q_A) \leq \pi_U(q_A) < \pi_U(q_U)$. □

Proof of Lemma 5. Assume $\gamma \in (0, 1)$. For $i = U, A$, the platform's first-order conditions are

$$\begin{cases} \frac{\partial \widehat{\Pi}_i}{\partial q} = (x_i\gamma + 1 - \gamma)\,\Pi_i'(q_i) + x_i\gamma\, s'(q_i) = 0, \\ \frac{\partial \widehat{\Pi}_i}{\partial x} = \gamma\left(\Pi_i(q_i) + s(q_i) + \widetilde{\xi}(x_i) + x_i\,\widetilde{\xi}'(x_i) - k\right) = 0. \end{cases} \tag{13}$$

Totally differentiating the equations in (13) with respect to γ, gives $\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2} q_i'(\gamma) + \frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q} x_i'(\gamma) + \frac{\partial^2 \widehat{\Pi}_i}{\partial \gamma \partial q} = 0$, and $\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} x_i'(\gamma) + \frac{\partial^2 \widehat{\Pi}_i}{\partial q \partial x} q_i'(\gamma) + \frac{\partial^2 \widehat{\Pi}_i}{\partial \gamma \partial x} = 0$. Moreover, evaluated at the equilibrium values q_i and x_i, $\frac{\partial^2 \widehat{\Pi}_i}{\partial \gamma \partial q} = -\Pi_i'(q_i) + x_i\left(\Pi_i'(q_i) + s'(q_i)\right)$ and $\frac{\partial^2 \widehat{\Pi}_i}{\partial \gamma \partial x} = 0$. Hence, solving the corresponding system for $q_i'(\gamma)$ and $x_i'(\gamma)$, yields

$$\begin{cases} q_i'(\gamma) = \left(-\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2}\right)\left[\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2}\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} - \left(\frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q}\right)^2\right]^{-1}\left[-\Pi_i'(q_i) + x_i\left(\Pi_i'(q_i) + s'(q_i)\right)\right], \\ x_i'(\gamma) = \left(\frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q}\right)\left[\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2}\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} - \left(\frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q}\right)^2\right]^{-1}\left[-\Pi_i'(q_i) + x_i\left(\Pi_i'(q_i) + s'(q_i)\right)\right]. \end{cases}$$

We now show that $q_i'(\gamma) > 0$. Recall that Second-Order Condition 5 in Appendix A.2 implies that $-\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} > 0$ and $\frac{\partial^2 \widehat{\Pi}_i}{\partial q^2}\frac{\partial^2 \widehat{\Pi}_i}{\partial x^2} - \left(\frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q}\right)^2 > 0$. Next, rewrite the first-order condition $\partial \widehat{\Pi}_i / \partial q = 0$ in (13) as $\Pi_i'(q_i) + \frac{\gamma x_i}{\gamma x_i + 1 - \gamma} s'(q_i) = 0$. Recall that $s'(q_i) > 0$. Also, since $x_i \in (0,1)$, we have $\frac{\gamma x_i}{\gamma x_i + 1 - \gamma} \in (0,1)$. Therefore, $\Pi_i'(q_i) + s'(q_i) > 0 > \Pi_i'(q_i)$ must hold. Consequently, $-\Pi_i'(q_i) + x_i\left(\Pi_i'(q_i) + s'(q_i)\right) > 0$. Finally, to show that $x_i'(\gamma) > 0$, note that $\frac{\partial^2 \widehat{\Pi}_i}{\partial x \partial q} = \gamma\left(\Pi_i'(q_i) + s'(q_i)\right) > 0$. This completes the proof. □

Proof of Lemma 6. First, note the following properties of the dependent share of purchasers.

(i) *Constant Boundaries:* for all $x \in [0,1]$, $\beta(x,0) = 0$, and for all $x \in (0,1]$, $\beta(x,1) = 1$;

(ii) *Equilibrium Continuity:* for all $\gamma \in [0,1]$, $\beta(x_i(\gamma), \gamma)$ is continuous with respect to γ. This follows from the fact that $\lim_{\gamma \to 0} \beta(x_i(\gamma), \gamma) = 0$, because $\lim_{\gamma \to 0} x_i(\gamma)$ is finite, and the fact that $\lim_{\gamma \to 1} \beta(x_i(\gamma), \gamma) = 1$, because $\lim_{\gamma \to 1} x_i(\gamma) \neq 0$.

(iii) *Equilibrium Monotonicity:* for all $\gamma \in (0,1)$, $\frac{d}{d\gamma}\{\beta(x_i(\gamma), \gamma)\} > 0$, $i = U, A$. This can be shown by expanding $\frac{d}{d\gamma}\{\beta(x_i(\gamma), \gamma)\}$ and noting, from Lemma 5, that $x_i'(\gamma) > 0$.

The claims of existence and uniqueness, as well as the ranking of γ_i^0 and γ_i^M, follow from the properties above and the fact that $\beta^0 < \beta_i^M$. To show the remaining claims: in part (a), note that the definition of γ_i^0 implies that, under both agreements $i = U, A$,

$$\begin{aligned}\frac{\partial \widehat{\Pi}_i\left(q^0, x_i(\gamma_i^0)\right)}{\partial q} = 0 \quad &\Leftrightarrow \quad \Pi_i'(q^0) + \frac{\gamma_i^0 x_i(\gamma_i^0)}{\gamma_i^0 x_i(\gamma_i^0) + 1 - \gamma_i^0} \cdot s'(q^0) = 0 \\ &\Leftrightarrow \quad \Pi_i'(q^0) + \beta\left(x_i(\gamma_i^0), \gamma_i^0\right) s'(q^0) = 0.\end{aligned}$$

$\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) = \beta^0$ then follows directly from Proposition 1.

In part (b), note that the definition of γ_i^0 implies that, under both agreements $i = U, A$,

$$\frac{\partial \widehat{\Pi}_i\left(q^M, x_i(\gamma_i^M)\right)}{\partial q} = 0 \quad \Leftrightarrow \quad \Pi_i'(q^M) + \beta\left(x_i(\gamma_i^M), \gamma_i^M\right) s'(q^M) = 0.$$

$\beta\left(x_i(\gamma_i^M), \gamma_i^M\right) = \beta_i^M$ then follows directly from Proposition 2. Moreover, since $\Pi_i(q^M) = 0$, it holds that $x_i(\gamma_i^M) = \operatorname{argmax}_x \left\{\gamma_i^M x\left(s(q^M) + \xi(x) - k\right)\right\} = x^M$. □

Proof of Lemma 7. For part (a) regarding a q^0-compatible market, first consider the case where $\beta^0 = 0$. Here, $\Pi_i'(q^0) = 0$, $i = U, A$, and thus $\gamma_U^0 = \gamma_A^0 = 0$. Next consider the case where $\beta^0 = 1$. Here, $\Pi_i'(q^0) + s(q^0) = 0$, $i = U, A$, and thus $\gamma_U^0 = \gamma_A^0 = 1$. (That is, for $i = U, A$, $\bar{q}_i = q^0$, where, recall from Appendix A.2, $\bar{q}_i \equiv \operatorname{argmax}_q \left\{\widehat{\Pi}_i(q,x)|_{\gamma=1}\right\}$.) Finally, consider the case where $\beta^0 \in (0,1)$. Here, $\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) \in (0,1)$, and $\gamma_i^0 \in (0,1)$, $i = U, A$. Hence, in this case, $\gamma_A^0 < \gamma_U^0 \Leftrightarrow x_A(\gamma_A^0) > x_U(\gamma_U^0)$.

We now show that this latter inequality must hold. To do so, note that, for $i = U, A$,

$$\frac{\partial\widehat{\Pi}_i\left(q^0, x_i(\gamma_i^0)\right)}{\partial x} = \left[\Pi_i(q^0) + s(q^0) + \xi\left(x_i(\gamma_i^0)\right) + x \cdot \xi'\left(x_i(\gamma_i^0)\right) - k\right]\gamma_i^0 = 0$$

Since $q^0 < q^M$, we have $\Pi_A(q^0) > \Pi_U(q^0)$. Moreover, due to Second-Order Condition 5, $\partial\widehat{\Pi}_i(q,x)/\partial x$ are strictly decreasing functions with respect to x, for both $i = U, A$. Thus, $x_A(\gamma_A^0) > x_U(\gamma_U^0)$.

For part (b), since the market is AM-compatible, we have $0 < \beta_U^M = \beta\left(x^M, \gamma_U^M\right) < \beta_A^M = \beta\left(x^M, \gamma_A^M\right) \leq 1$. Also, for $i = U, A$, we have $x_i(\gamma_i^M) = x^M$. Therefore, $\gamma_U^M < \gamma_A^M$. □

Proof of Proposition 4. This result follows directly from the definition of γ_i^M, the fact that $q_i'(\gamma) > 0$, shown in Lemma 5, and the fact that $\gamma_U^M < \gamma_A^M$, shown in Lemma 7. □

Proof of Proposition 5. For part (a), first consider the case where $\gamma = 0$. We then have $x_i = 0$, $i = U, A$ (see footnote 24). If $\gamma_U^0 = 0$, then $\beta^0 = 0$, implying $q_i = q^0$, $i = U, A$. If $\gamma_U^0 \in (0,1)$, then $\beta^0 \in (0,1)$ and thus, when $\gamma = 0$, Proposition 1 implies $q^0 > q_A > q_U$.

Second, consider the case where $\gamma \in (0, \gamma_U^0]$. Note that $\gamma_U^0 < \gamma_A^M \leq 1$. Here, it suffices to show (i), (ii) and (iii).

(i) $\left.\frac{\partial\widehat{\Pi}_A}{\partial q}\right|_{(q_U, x_U)} = (\gamma x_U + 1 - \gamma)\,\Pi_A'(q_U) + x_U\,\gamma\, s'(q_U) \geq 0.$

(ii) $\left.\frac{\partial\widehat{\Pi}_A}{\partial x}\right|_{(q_U, x_U)} = \left(\Pi_A(q_U) + s(q_U) + \widetilde{\xi}(x_U) + x_U\,\widetilde{\xi}'(x_U) - k\right)\gamma > 0.$

(iii) $\frac{\partial^2\widehat{\Pi}_A}{\partial q\partial x} = \left(\Pi_A'(q) + s'(q)\right)\gamma > 0$, for all q in the closed interval with endpoints q_U and q_A.

For (i), recall that Proposition 1 shows that, for a given $\beta \leq \beta^0$, if q^* satisfies $\Pi_U'(q^*) + \beta s'(q^*) = 0$, then $\Pi_A'(q^*) + \beta s'(q^*) \geq 0$. Now note that q_U satisfies $\Pi_U'(q_U) + \beta\left(x_U(\gamma), \gamma\right) s'(q_U) = 0$, and the inequality we want to show is equivalent to $\Pi_A'(q_U) + \beta\left(x_U(\gamma), \gamma\right) s'(q_U) \geq 0$. Hence, it suffices to show that $\beta\left(x_U(\gamma), \gamma\right) \leq \beta^0$. This holds, because $d\beta(x_U(\gamma), \gamma)/d\gamma > 0$ (Equilibrium Monotonicity established in the proof of Lemma 6) and because $\gamma \leq \gamma_U^0$.

For (ii), since $\gamma \leq \gamma_U^0$, we have $q_U \leq q^0 < q^M$ and thus $\Pi_A(q_U) > \Pi_U(q_U)$. Under a U-agreement, $\left.\frac{\partial\widehat{\Pi}_U}{\partial x}\right|_{(q_U, x_U)} = \left(\Pi_U(q_U) + s(q_U) + \widetilde{\xi}(x_U) + x_U\,\widetilde{\xi}'(x_U) - k\right)\gamma = 0$, and so we are done.

For (iii), recall from Appendix A.2 that $\overline{q}_i \equiv \text{argmax}_q\left\{\widehat{\Pi}_i(q,x)|_{\gamma=1}\right\}$, and thus $\Pi_A'(\overline{q}_A) + s'(\overline{q}_A) = 0$. Furthermore, Second-Order Condition 5 implies that, for all $q < \overline{q}_A$, we have $\Pi_A'(q) + s'(q) > 0$. Hence, it suffices to show that for both $i = U, A$, $q_i < \overline{q}_A$. Recall that, for $i = U, A$, $q_i'(\gamma) > 0$. First, for $i = U$, since $\gamma \leq \gamma_U^0$, we have $q_U \leq q^0$, and since $\gamma_A^0 < 1$, we have $q^0 < \overline{q}_A$. Therefore, we have $q_U < \overline{q}_A$. Second, for $i = A$, since $\gamma \leq \gamma_U^0 < 1$, we have $q_A < \overline{q}_A$. This completes the proof of (a).

For part (b), we first show that, if $\gamma \in [\gamma_U^M, 1]$, then $q_U > q_A$ and $x_U > x_A$. Then we show that γ^x exists, lies in (γ_U^0, γ_U^M), and that, when $\gamma \in (\gamma^x, \gamma_U^M)$, the same inequalities hold.

If $\gamma \in [\gamma_U^M, \gamma_A^M]$, then $q_U > q_A$ and $x_U > x_A$, because, by definition, $q_U(\gamma_U^M) = q_A(\gamma_A^M) = q^M$, from Lemma 6 we have $x_U(\gamma_U^M) = x_A(\gamma_A^M) = x^M$, and $q_i'(\gamma), x_i'(\gamma) > 0$, $i = U, A$.

If $\gamma_A^M < 1$ and $\gamma \in (\gamma_A^M, 1]$, then, analogously to (a), it suffices to show (iv), (v) and (vi).

(iv) $\left.\frac{\partial \widehat{\Pi}_U}{\partial q}\right|_{(q_A, x_A)} = (\gamma x_A + 1 - \gamma)\, \Pi'_U\,(q_A) + x_A\, \gamma\, s'\,(q_A) > 0.$

(v) $\left.\frac{\partial \widehat{\Pi}_U}{\partial x}\right|_{(q_A, x_A)} = \left(\Pi_U(q_A) + s(q_A) + \widetilde{\xi}(x_A) + x_A\, \widetilde{\xi}'(x_A) - k\right)\gamma > 0.$

(vi) $\frac{\partial^2 \widehat{\Pi}_U}{\partial q \partial x} = \left(\Pi'_U\,(q) + s'\,(q)\right)\gamma \geq 0$, for all q in the closed interval with endpoints q_A and q_U.

For (iv), recall that Proposition 1 shows that, for a given $\beta > \beta^0$, if q^* satisfies $\Pi'_A(q^*) + \beta s'(q^*) = 0$, then $\Pi'_U(q^*) + \beta s'(q^*) \geq 0$. Now note that $\gamma > \gamma_A^0$ and thus $\beta\,(x_A(\gamma), \gamma) > \beta^0$. A symmetric argument to the one given for (i) then shows that the desired inequality holds.

For (v), since $\gamma > \gamma_A^M$, we have $q_A > q^M$ and thus $\Pi_U(q_A) > \Pi_A(q_A)$. Under an A-agreement, $\left.\frac{\partial \widehat{\Pi}_A}{\partial x}\right|_{(q_A, x_A)} = \left(\Pi_A(q_A) + s(q_A) + \widetilde{\xi}(x_A) + x_A\, \widetilde{\xi}'(x_A) - k\right)\gamma = 0$, and so we are done.

For (vi), following the logic of (iii) it suffices to show that for both $i = U, A$, $q_i \leq \overline{q}_U$. Since for $i = U, A$, $q'_i(\gamma) > 0$, we have $q_i \leq \overline{q}_i$. Thus it remains only to show $\overline{q}_A \leq \overline{q}_U$. Since, for all $x \in (0, 1]$, we have $\beta(x, 1) = 1 > \beta^0$, it follows from Proposition 1 that $\overline{q}_A < \overline{q}_U$. Thus we have shown that, if $\gamma \in [\gamma_U^M, 1]$, then $q_U > q_A$ and $x_U > x_A$.

The existence of γ^x and its value in the interval (γ_U^0, γ_U^M) follows from the fact that $x_i(\gamma)$, $i = U, A$, are continuous functions, and, as shown above, $x_A(\gamma_U^0) > x_U(\gamma_U^0)$ and, for all $\gamma \in [\gamma_U^M, 1]$, $x_U(\gamma) > x_A(\gamma)$. These facts also imply that, if $\gamma \in (\gamma^x, \gamma_U^M)$, then $x_U > x_A$.

It remains only to show that, if $\gamma \in (\gamma^x, \gamma_U^M)$, then $q_U > q_A$. For this, it suffices to note that, since $\gamma > \gamma^x > \gamma_U^0$ and $x_U > x_A$, we have $\beta\,(x_U(\gamma), \gamma) > \beta\,(x_A(\gamma), \gamma) > \beta^0$. Proposition 1 and the fact that both $q'_U(\gamma) > 0$ and $d\beta/d\gamma > 0$ then imply that $q_U > q_A$. This completes the proof. □

Proof of Proposition 6. For part (a), we begin by showing that (i) $\gamma \in [0, \gamma_U^M] \Rightarrow \widehat{\Pi}_A(q_A, x_A) > \widehat{\Pi}_U(q_U, x_U)$ and (ii) $\gamma \in [\gamma_A^M, 1] \Rightarrow \widehat{\Pi}_U(q_U, x_U) > \widehat{\Pi}_A(q_A, x_A)$. We then show (iii) that, over the interval (γ_U^M, γ_A^M), these two functions cross exactly once. Recall from equation (9) that, for $i = U, A$, $\widehat{\Pi}_i(q, x) = \gamma x \left(\Pi_i(q) + s(q) + \widetilde{\xi}(x) - k\right) + (1 - \gamma)\Pi_i(q)$.

For (i), assume $\gamma \in [0, \gamma_U^M]$. This implies that $q_U \leq q_M$ and thus $\Pi_U(q_U) \leq \Pi_A(q_U)$. Hence, $\widehat{\Pi}_U(q_U, x_U) \leq \widehat{\Pi}_A(q_U, x_U) < \widehat{\Pi}_A(q_A, x_A)$. Analogously for (ii), $\gamma \in [\gamma_A^M, 1] \Rightarrow q_A \geq q_M \Rightarrow \Pi_A(q_A) \leq \Pi_U(q_A)$. Hence, $\widehat{\Pi}_A(q_A, x_A) \leq \widehat{\Pi}_U(q_A, x_A) < \widehat{\Pi}_U(q_U, x_U)$.

For (iii), it suffices to show that, throughout the interval (γ_U^M, γ_A^M),

$$\frac{d}{d\gamma}\left\{\widehat{\Pi}_A - \widehat{\Pi}_U\right\} < 0. \tag{14}$$

Note, from Proposition 4, that, in this interval, the platform taxes the good under an A-agreement and subsidizes it under a U-agreement, and thus $q_A < q_M < q_U$, implying $\Pi_A(q_A) > 0 > \Pi_U(q_U)$.

Using the envelope theorem as in the proof of Proposition 3(b), we have, for $i = U, A$, $\frac{d\widehat{\Pi}_i}{d\gamma} = \frac{\partial \widehat{\Pi}_i}{\partial \gamma} = -(1 - x_i)\Pi_i(q_i) + x_i\left(s(q_i) + \widetilde{\xi}(x_i) - k\right)$. Plugging this in for $i = U, A$ and rearranging, the condition in (14) becomes

$$\underbrace{[(1-x_U)\Pi_U(q_U)-(1-x_A)\Pi_A(q_A)]}_{<0} \;+\; x_A\left(t(q_A,x_A)-k\right) \;-\; x_U\left(t(q_U,x_U)-k\right)<0. \tag{15}$$

Note that the second and third terms in (15) are (proportional to) the platform's direct profits from the sale of access, under an A- and a U-agreement, respectively. These direct profits from access must be strictly greater under the regime in which the platform subsidizes than under the regime in which it taxes. To prove this, note that, by revealed preference,

$$\begin{aligned}
&\widehat{\Pi}_U(q_U,x_U)>\widehat{\Pi}_U(q_A,x_A)\\
\Leftrightarrow\;&(x_U\gamma+1-\gamma)\Pi_U(q_U)+\gamma x_U\left(t(q_U,x_U)-k\right)>(x_A\gamma+1-\gamma)\Pi_U(q_A)+\gamma x_A\left(t(q_A,x_A)-k\right)\\
\Rightarrow\;&x_U\left(t(q_U,x_U)-k\right)>x_A\left(t(q_A,x_A)-k\right).
\end{aligned}$$

Therefore, the condition in (14) is indeed satisfied.

For part (b), since $\gamma\in[\gamma^x,\gamma_A^M]$, we have both $q_A\le q^M$ and $q_A<q_U$. Thus, following the proof of Proposition 3(b), we have $\pi_U(q_U)>\pi_A(q_A)$. In addition, since $x_U\ge x_A$, *a fortiori*, we have $\hat{\pi}_U(q_U,x_U)>\hat{\pi}_A(q_A,x_A)$. □

Proof of Proposition 7. The claim follows from the fact that

$$\left[\gamma^{\widehat{\Pi}},\gamma_A^M\right]=\underbrace{[\gamma^x,1]}_{\text{Prop. 5(b)}}\;\cap\;\underbrace{\left[\gamma^{\widehat{\Pi}},1\right]}_{\text{Prop. 6(a)}}\;\cap\underbrace{\left[\gamma^x,\gamma_A^M\right]}_{\text{Prop. 6(b)}}.$$

□

C Extensions and Additional Examples

C.1 Platform Marginal Costs

Consider an environment identical to the one described in Section II, but where the platform incurs marginal cost $\omega\ge 0$ for each unit of the good that is sold. (In Section II, we assume $\omega=0$.) All of the results stated in that section continue to hold, but where the profit-neutral and no-markup values of β are shifted up by $\omega/s'(q^0)$ and $\omega/s'(q^M)$, respectively. Note, in particular, that the values of q^0 and q^M are not affected and remain key building blocks. Specifically, retaining the mathematical definitions of β^0, β_U^M and β_A^M, as they arise from demand for the good, $q(p)$, and the producer's marginal cost, c, the generalized results are as follows.

Proposition 1′. *If $\beta<\beta^0+\omega/s'(q^0)$, then $q_U<q_A<q^0$; if $\beta=\beta^0+\omega/s(q^0)$, then $q_U=q_A=q^0$; and if $\beta>\beta^0+\omega/s'(q^0)$, then $q_U>q_A>q^0$. Therefore, the good's price is lower under a U-agreement than under an A-agreement whenever β is greater than $\beta^0+\omega/s'(q^0)$.*

Proposition 2′. *Under an agreement of type $i = U, A$, when $\beta = \beta_i^M + \omega/s'(q^M)$, it is optimal for the platform to neither tax nor subsidize the good, inducing the producer to choose q^M. Furthermore, (a) if $\beta < \beta_U^M + \omega/s'(q^M)$, then the platform taxes under both forms of agreement ($r, a > 0$); (b) if $\beta \in \left(\beta_U^M + \omega/s'(q^M), \beta_A^M + \omega/s'(q^M)\right)$, then the platform subsidizes under a U-agreement and taxes under an A-agreement ($r < 0 < a$); (c) if $\beta > \beta_A^M + \omega/s'(q^M)$, then the platform subsidizes under both forms of agreement ($r, a < 0$). No primitives exist that lead the platform to tax in a U-agreement but subsidize in an A-agreement.*

Proposition 3′. *(a) There exists a unique $\beta^{\widehat{\Pi}}$, and it lies in the interval $\left(\beta_U^M + \omega/s'(q^M), \beta_A^M + \omega/s'(q^M)\right)$. Moreover, $\beta > \beta^{\widehat{\Pi}} \Leftrightarrow \widehat{\Pi}_U(q_U) > \widehat{\Pi}_A(q_A)$. (b) $\beta \in \left[\beta^0 + \omega/s(q^0), \beta_A^M + \omega/s(q^M)\right] \Rightarrow \pi_U(q_U) > \pi_A(q_A)$. Therefore, whenever $\beta \in \left(\beta^{\widehat{\Pi}}, \beta_A^M + \omega/s'(q^M)\right]$, switching from an A-agreement to a U-agreement increases the profits of the platform and the producer and raises consumer surplus ($s(q_U) > s(q_A)$).*

The sketch of the proofs for these propositions is as follows. (The formal proofs of Propositions **1′**, **2′**, and **3′** follow closely those of Propositions 1, 2, and 3 in the main text, respectively, and are thus omitted.) First, note that the model is similar to that described in Section II. The only change in the model from the main text is that the platform's total profit under a U-agreement is now given by $r \cdot q + \beta \cdot s(q) - \omega \cdot q$, and it is $a \cdot p \cdot q + \beta \cdot s(q) - \omega \cdot q$ under an A-agreement. The producer's profits remain unchanged. Thus, following Lemma 1, under both forms of agreement, the equilibrium quantity, q_i, satisfies

$$mr(q_i) - c = -\beta s'(q_i) + \omega + \pi_i'(q_i) = \left[-\beta + \omega/s'(q_i)\right] s'(q_i) + \pi_i'(q_i), \tag{16}$$

where $\pi_U'(q) = -mr'(q)\, q$ and $\pi_A'(q) = -mr'(q)\, q \frac{p(q)/mr(q)}{mr(q)/c}$, for $i = U, A$. Therefore, any relevant threshold over β governing our results is *shifted* by $\omega/s'(q_i) \geq 0$ when the platform's marginal cost is positive. Importantly, this holds for the thresholds indicating (i) whether a U- or an A-agreement leads to a lower price for the goods (Proposition **1′**), (ii) whether U- and/or A-agreements provide the platform with the incentives to "tax" or to "subsidize" the goods (Proposition **2′**), and (iii) whether a U- or an A-agreement is more profitable for the platform (Proposition **3′**).

The results above imply that, all else equal, greater platform externalities are required to reach a given threshold, for a given demand form, when the platform faces positive marginal costs. Consider, for instance, the relevant threshold for comparing consumer prices across agreements, under Example 1's constant elasticity specification (for which $\beta^0 = 0$). According to Proposition 1 in the main text, when $\omega = 0$, we obtain $q_A = q_U$ when $\beta = 0$, and $q_A < q_U$ when $\beta > 0$. However, if $\omega > 0$, Proposition **1′** above implies that when $\beta = 0$ we have $q_A > q_U$, and that $q_A < q_U$ holds only for large enough platform externalities (i.e., $\beta > \omega/s(q^0)$).[29]

Consequently, the size of a platform's marginal cost is potentially significant in influencing

[29]This point about platform marginal costs explains why, in its constant elasticity demand model with no element of user access, Shy and Wang (2011) finds an A-agreement yields a lower good price than a U-agreement.

which of our results are most applicable, for a given industry. Indeed, when the good has a low marginal cost – for instance, in digital markets – we expect the results to be economically approximated by the version in the main text. However, in physical settings where the platform's marginal costs can be substantial, this may have a sizeable effect on the relevant thresholds and, thus, on the level of platform externalities needed to reach the various ranges characterized in our propositions.

Finally, in the Section III environment, generalized to allow for $\omega \geq 0$, the results also continue to hold, with adjustments similar to those described above. Specifically, since, in this setting, $\Pi_U(q^M) = \Pi_A(q^M) = -\omega q^M$, we have $x^M \equiv \text{argmax}_x \left\{\left(t(q^M, x) - \omega q^M - k\right) x\right\}$, which is still invariant with respect to the form of agreement. For $i = U, A$, the threshold values of γ are as follows. γ_i^0 is such that $\beta\left(x_i(\gamma_i^0), \gamma_i^0\right) = \beta^0 + \omega/s'(q^0)$, and γ_i^M is such that $\beta\left(x_i(\gamma_i^M), \gamma_i^M\right) = \beta_i^M + \omega/s'(q^M)$, where it still holds that $x_i(\gamma_i^M) = x^M$.

C.2 General Externalities from the Market for the Good

We now generalize Section II's approach to modeling user access, showing that, in this more abstract environment, both the basic patterns identified in the main text still hold, and the same set of statistics from the good market that we identify as crucial retain their importance. In particular, here we dispense with the assumption that the externality perceived by the platform from the good market need be a constant proportion of users' surplus from that market; instead, this externality is simply captured by a general function of the quantity of the good that is sold.[30]

Consider that the platform externalities from selling the good are now represented by a function $\phi(\cdot)$, which takes the quantity of the good sold, q, as an argument, and which is twice differentiable. As above, c denotes the producer's marginal cost of selling the usage good. The rest of the model is similar to that described in Section II. The main difference with the simple model from that section is thus that, when selling q units of the good, the platform externalities are now represented by $\phi(q)$ instead of $\beta \cdot s(q)$. In particular, the platform's total profits under a U-agreement is now given by $r \cdot q + \phi(q)$, whereas it is equal to $a \cdot p \cdot q + \phi(q)$ under an A-agreement. The producer's profits remain unchanged.

Note, in particular, that the values of q^0 and q^M and the mathematical definitions of β^0, β_U^M and β_A^M are not affected by such externalities. We obtain the following results.

Proposition 1″. *If $\phi'(q^0)/s'(q^0) < \beta^0$, then $q_U < q_A < q^0$; if $\phi'(q^0)/s'(q^0) = \beta^0$, then $q_U = q_A = q^0$; and if $\phi'(q^0)/s'(q^0) > \beta^0$, then $q_U > q_A > q^0$. Therefore, the good's price is lower under a U-agreement than under an A-agreement whenever $\phi'(q^0)/s'(q^0)$ is greater than β^0.*

Proposition 2″. *Under an agreement of type $i = U, A$, when $\phi'(q^M)/s'(q^M) = \beta_i^M$, it is optimal for the platform to neither tax nor subsidize the good, inducing the producer to choose q^M. Furthermore, (a) if $\phi'(q^M)/s'(q^M) < \beta_U^M$, then the platform taxes under both forms of agreement ($r, a > 0$); (b) if*

[30] This exercise contrasts with Section III in that the latter fully specifies a model that boils down precisely to the one in Section II with one of the platform's choice variables (x) frozen.

$\phi'(q^M)/s'(q^M) \in \left(\beta_U^M, \beta_A^M\right)$, then the platform subsidizes under a U-agreement and taxes under an A-agreement ($r < 0 < a$); (c) if $\phi'(q^M)/s'(q^M) > \beta_A^M$, then the platform subsidizes under both forms of agreement ($r, a < 0$). No primitives exist that lead the platform to tax in a U-agreement but subsidize in an A-agreement.

The sketch of the proofs is as follows. (The formal proofs of Propositions **1″** and **2″** follow closely that from the corresponding propositions in the main text and are thus omitted.) Following Lemma 1, under both forms of agreement, the equilibrium quantity, q_i, satisfies

$$mr\,(q_i) - c = -\phi'\,(q_i) + \pi_i'\,(q_i)\,, \tag{17}$$

where $\pi_U'\,(q) = -mr'\,(q)\,q$ and $\pi_A'\,(q) = -mr'\,(q)\,q\frac{p(q)/mr(q)}{mr(q)/c}$, for $i = U, A$. We also assume that the corresponding second-order conditions hold. Defining $U_\phi\,(q) \equiv \phi'\,(q) + mr\,(q) + q\,mr'\,(q)$ and $A_\phi\,(q) \equiv \left(\phi'\,(q) + mr\,(q)\right) / \left(1 - q\,mr'\,(q)\;p\,(q)\,/mr\,(q)^2\right)$, these are equivalent to $U_\phi'\,(q) < 0$ for all q such that $U_\phi\,(q) > 0$ and $A_\phi'\,(q) < 0$ for all q such that $A_\phi\,(q) > 0$. These conditions ensure that $q_i < q^0 \Leftrightarrow \phi'(q^0)/s'(q^0) < \beta^0$ and $q_i < q^M \Leftrightarrow \phi'(q^M)/s'(q^M) < \beta_i^M$, $i = U, A$. Following the proofs of Propositions 1 and 2 in the main text, we thus see that the relevant ratio for comparing the effects of U- and A-agreements is $\phi'\,(\cdot)\,/s'\,(\cdot)$, evaluated at q^0 and q^M.

Moreover, we can state the following analog to Proposition 3(a) establishing the platform's preference for A-agreements in situations where it taxes and U-agreements in situations where it subsidizes. Unlike its analog in the main text, in this setting, we cannot guarantee a "single crossing" of the two profit functions, due to the lack of a suitable parameter to vary independently. (The formal proofs of Proposition **3″** follows closely that from the corresponding proposition in the main text and is thus omitted.)

Proposition 3″. *If $\phi'(q^M)/s'(q^M) \leq \beta_U^M$, then the platform earns greater total profits under an A-agreement; if $\phi'(q^M)/s'(q^M) \geq \beta_A^M$, then the platform earns greater total profits under a U-agreement.*

C.3 Derivations for Subsection III.D: Markets with More General Demand

C.3.1 Strictly Log-Concave Demand

Assume that demand is globally strictly log-concave, i.e., $\sigma(q) < 1 \Leftrightarrow \delta(q) < 1$, $\forall q$. In this case, $\beta_A^M > \beta_U^M > 1$, and so demand is neither *AM*- nor *UM*-compatible. In this environment, the arguments used in Propositions 4 and 6 imply, respectively, that

- the platform taxes the good under both forms of agreement: $r, a > 0$;
- the platform's profits are greater under an A-agreement: $\widehat{\Pi}(q_A, x_A) > \widehat{\Pi}(q_U, x_U)$.

To compare q's and x's, further assume that demand falls into one of the following two subcases.

(a) $\sigma(q) < \frac{1}{\varepsilon(q)} \Leftrightarrow \frac{-q\varepsilon'(q)}{\varepsilon(q)} > 1$, $\forall q \Rightarrow \beta^0 > 1$, and so demand is not q^0-compatible. For all $\gamma \in [0,1]$, $q_A > q_U$ and $x_A > x_U$. Therefore, all consumers are strictly better off under an A-agreement, except those who are inactive under both forms.

(b) $\sigma(q) \geq \frac{1}{\varepsilon(q)} \Leftrightarrow \frac{-q\varepsilon'(q)}{\varepsilon(q)} \in (0,1]$, $\forall q \Rightarrow \beta^0 \in (0,1]$, and so demand is q^0-compatible. If $\gamma \in (0,\gamma_U^0]$, then $q_A > q_U, x_A > x_U$, and all consumers are strictly better off under an A-agreement, except those who are inactive under both forms.

In both subcases, the claims follow from the argument used in Proposition 5(a).

C.3.2 Strictly Super-Convex Demand

Assume that demand is globally strictly super-convex, i.e., $\sigma(q) > 1 + \frac{1}{\varepsilon(q)} \Leftrightarrow \varepsilon'(q) > 0$, $\forall q$. In this case, $\beta^0 < 0 < \beta_U^M < \beta_A^M < 1$, and so demand is not q^0-compatible, but it is *UM*-compatible and *AM*-compatible. In this environment:

- Propositions 4, 6(a), and 7 hold, exactly as stated in the main text;
- Propositions 5(b) and 6(a) are substantively unchanged; however, γ^x may not exist, and when this is the case, the conclusions hold, respectively, for all $\gamma \in [0,1]$ and $\gamma \in \left[0,\gamma_A^M\right]$;
- Proposition 5(a), is not applicable, because γ_U^0 does not exist.

C.3.3 Log-Convex yet Strictly Sub-Convex Demand

Assume that demand is globally log-convex and strictly sub-convex, i.e., $1 \leq \sigma(q) < 1 + \frac{1}{\varepsilon(q)}$, $\forall q$. In this case, $0 < \beta^0 < \beta_U^M \leq 1 < \beta_A^M$, and so demand is q^0-compatible and *UM*-compatible but not *AM*-compatible. In this environment, the following results hold due to arguments that are unchanged.

- As in Proposition 4, if $\gamma \in [0,\gamma_U^M)$, then $r, a > 0$, and if $\gamma \in (\gamma_U^M, 1]$, then $r < 0 < a$.
- As in Proposition 5(a), if $\gamma \in [0,\gamma_U^0]$, then $q_A > q_U$, $x_A \geq x_U$, where the latter inequality is strict unless $\gamma = 0$, and all consumers are strictly better off under an A-agreement, except those who are inactive under both forms.
- As in Proposition 6(a), if $\gamma \in [0,\gamma_U^M]$, then $\widehat{\Pi}_A(q_A,x_A) > \widehat{\Pi}_U(q_U,x_U)$, and if $\gamma \in (\gamma_U^M, 1)$, then $\frac{d}{d\gamma}\left\{\widehat{\Pi}_A - \widehat{\Pi}_U\right\} < 0$.

Among the class of demand functions we are considering, those that are sufficiently convex feature $\beta^{\widehat{\Pi}} \leq 1$, whereas those that are sufficiently close to the boundary with log-concavity have $\beta^{\widehat{\Pi}} > 1$. A profit-neutral threshold, $\gamma^{\widehat{\Pi}} \in (\gamma_U^M, 1]$, exists if and only if $\beta^{\widehat{\Pi}} \leq 1$. When this condition is satisfied, we can state modified versions of the remaining results.

Proposition 5(b)′. *If* $\beta^{\widehat{\Pi}} \leq 1$, *then* *(i)* γ^x *exists and lies in the interval* $(\gamma_U^0, \gamma^{\widehat{\Pi}})$;

(ii) *if* $\gamma \in [\gamma^x, 1]$, *then* $q_U > q_A$ *and* $x_U \geq x_A$, *where the second inequality is strict unless* $\gamma = \gamma^x$.

Therefore, in this case, all consumers are weakly better off under a U-agreement.

Proof. For part (i), note, first, that the argument in Proposition 5(a) implies that, whenever $\gamma \leq \gamma_U^0$, $x_A > x_U$. We now show that, whenever $\gamma \geq \gamma^{\widehat{\Pi}}$, $x_U > x_A$.

If $\gamma \geq \gamma^{\widehat{\Pi}}$, then, since $\frac{d}{d\gamma}\left\{\widehat{\Pi}_A - \widehat{\Pi}_U\right\} < 0$, it holds that $\widehat{\Pi}_U(q_U, x_U) \geq \widehat{\Pi}_A(q_A, x_A) \Leftrightarrow$

$$(1-\gamma)\Pi_U(q_U) + \gamma x_U \left(\Pi_U(q_U) + s(q_U) + \widetilde{\xi}(x_U) - k\right) \\ \geq (1-\gamma)\Pi_A(q_A) + \gamma x_A \left(\Pi_A(q_A) + s(q_A) + \widetilde{\xi}(x_A) - k\right). \quad (18)$$

Since $\Pi_U(q_U) < 0 < \Pi_A(q_A)$, (18) implies that

$$x_U\left(\widetilde{\xi}(x_U) - [k - (\Pi_U(q_U) + s(q_U))]\right) > x_A\left(\widetilde{\xi}(x_A) - [k - (\Pi_A(q_A) + s(q_A))]\right). \quad (19)$$

Second-Order Condition 5(ii) implies that both $\operatorname{argmax}_x x\left(\widetilde{\xi}(x) - \kappa\right)$ and $\max_x x\left(\widetilde{\xi}(x) - \kappa\right)$ strictly decrease as κ increases. (That is, under a well-behaved monopoly problem, both output and profits decrease as marginal cost increases.) Since, in the respective expressions in (19), for $i = U, A$, $\Pi_i(q_i) + s(q_i)$ enters additively as a shifter of the marginal cost of providing access, this inequality implies both that $\Pi_U(q_U) + s(q_U) > \Pi_A(q_A) + s(q_A)$ and that $x_U > x_A$.

Since $x_A(\gamma_U^0) > x_U(\gamma_U^0)$, since, for all $\gamma \in [\gamma^{\widehat{\Pi}}, 1]$, $x_U(\gamma) > x_A(\gamma)$, and since the functions $x_i(\cdot)$, $i = U, A$, are continuous, we have that γ^x exists and lies in the interval $(\gamma_U^0, \gamma^{\widehat{\Pi}})$.

For part (ii), the claim that $x_U > x_A$ has already been established in the preceding argument. To show that $q_U > q_A$, it suffices to note that, since $\gamma > \gamma^x > \gamma_U^0$, we have $x_U > x_A$ and thus $\beta(x_U(\gamma), \gamma) > \beta(x_A(\gamma), \gamma) > \beta^0$. Proposition 1 and the fact that both $q_U'(\gamma) > 0$ and $d\beta/d\gamma > 0$ then imply that $q_U > q_A$. □

Given this argument, the remaining, modified propositions follow in a straightforward way.

Proposition 6′. *If* $\beta^{\widehat{\Pi}} \leq 1$, *then*

(a) $\gamma > \gamma^{\widehat{\Pi}} \Leftrightarrow \widehat{\Pi}_U(q_U, x_U) > \widehat{\Pi}_A(q_A, x_A)$.

(b) $\gamma \in [\gamma^x, 1] \Rightarrow \hat{\pi}_U(q_U) > \hat{\pi}_A(q_A)$.

Proposition 7′. *If* $\beta^{\widehat{\Pi}} \leq 1$, *then whenever* $\gamma \in \left[\gamma^{\widehat{\Pi}}, 1\right]$, *a U-agreement Pareto-dominates an A-agreement.*

C.4 Additional Examples of Demand

Example 3 ("Generalized Pareto" Demand)**.** *Let*

$$q(p) = \left(1 + \frac{\lambda(\sigma-1)(p-c)}{c}\right)^{\frac{1}{1-\sigma}},$$

where $\lambda > 0$, $\sigma \in (1, 2)$, and $c > 0$. Here, σ directly parameterizes the (constant) curvature of demand, and c denotes, as in the main text, the producer's marginal cost. By construction, this function is defined over the domain $[c, \infty)$, with $q(c) = 1$, and $\lim_{p\to\infty} = 0$.[31] *This gives rise to inverse demand of*

$$p(q) = \left(\frac{q^{1-\sigma} - 1}{\lambda(\sigma - 1)} + 1 \right) c,$$

defined over the domain $(0, 1]$.

This specification, borrowed from Wang and Wright (2017, 2018), is a re-parameterization of a general class appearing in Bulow and Klemperer (2012). For our purposes, a notable feature of this demand form is that when $\sigma \in (1, 1 + 1/\lambda)$, it is strictly log-convex yet sub-convex, and when $\sigma \in (1 + 1/\lambda, 2)$ it is strictly super-convex. (When $\sigma = 1 + 1/\lambda$, it is the special case of the constant elasticity demand of Example 1 in which $\varepsilon = \lambda$ and $\alpha = c$.)

Recall from Subsection III.D and Appendix C.3.3 that a key issue for markets with log-convex yet sub-convex demand is whether or not $\beta^{\widehat{\Pi}} \leq 1$. Under this demand form, $\beta_U^M = 2 - \sigma$ and $\beta_A^M = 2 - \sigma + \frac{1}{\lambda}$, and, Proposition 3(a) shows, $\beta^{\widehat{\Pi}} \in \left(\beta_U^M, \beta_A^M\right)$. Therefore, a threshold curvature level exists, somewhere in the interval $(1, 1 + 1/\lambda)$, such that when σ is below this level, $\beta^{\widehat{\Pi}} > 1$ and when σ is above this level, $\beta^{\widehat{\Pi}} < 1$. In the latter case, Propositions **5(b)′**, **6′**, and **7′** all apply. Figure 3 plots all of the relevant thresholds in this example.

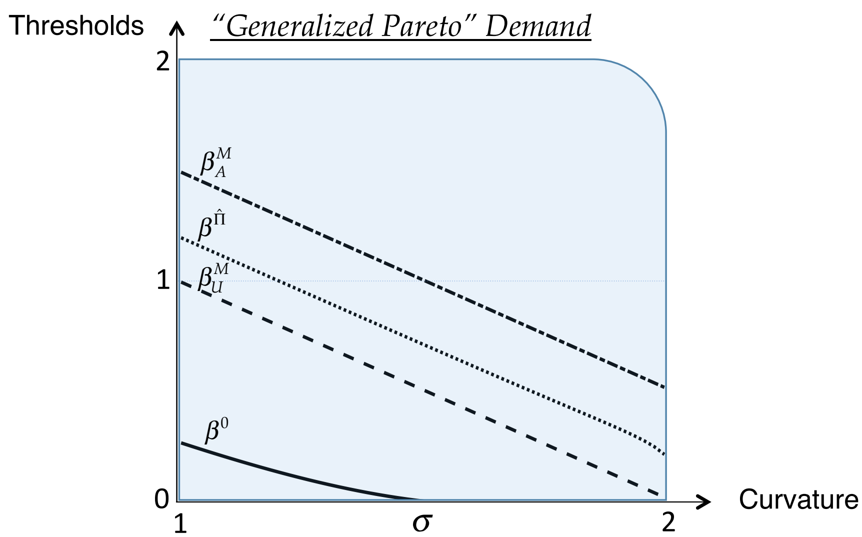

Figure 3: *Thresholds as functions of curvature in Example 3.*[32]

Example 4 (Constant β^0)**.** *Let $p(q) = \Theta \exp\left[-\Lambda/B \cdot q^B\right]$, where $\Theta, \Lambda > 0$ and $B \neq 0$.*[33]

[31] The output of the function should be interpreted as the ratio: "quantity of the good a consumer purchases given price $p \geq c$" / "quantity she would purchase if it were priced at the seller's marginal cost."

[32] In Figure 3, $\lambda = 2$ and $c = 1$. In Example 3, $\beta^0 = \left[-2/\left(1 + \sqrt{1 - 4(2-\sigma)(\sigma - 1 - \frac{1}{\lambda})}\right) + 1\right](2 - \sigma)$.

[33] We thank an anonymous referee for pointing out this specification to us.

Under this specification, studied by Klenow and Willis (2016) and discussed by Mràzovà and Neary (2017), for all $q \geq 0$, it holds that $-q\varepsilon'(q)/\varepsilon(q) = B$. Thus, parameter B directly captures the quantity-neutral level of user-access, $\beta^0 \equiv -q^0\varepsilon'(q^0)/\varepsilon(q^0)$, established in Definition 3. Therefore, in this case, the price comparison across the two forms of agreement, on which Proposition 1 focuses, can be made by simply comparing B and β, without the need to compute q^0. Similarly, with this form of demand, $\beta(x_i, \gamma) \leq B \Leftrightarrow \gamma \leq \gamma_i^0$, $i = U, A$. Consequently, if $\beta(x_U, \gamma) \leq B$, then the conclusion in Proposition 5(a) holds.

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